# ELECTRICAL PROBLEMS

FOR

## *ENGINEERING STUDENTS*

BY

WILLIAM L[eslie]. HOOPER, PH. D.
PROFESSOR OF ELECTRICAL ENGINEERING, TUFTS COLLEGE, MASS.

AND

ROY T. WELLS, M.S.
SENIOR FELLOW IN PHYSICS, CLARK UNIVERSITY, WORCESTER, MASS.

BOSTON, U.S.A., AND LONDON
GINN & COMPANY, PUBLISHERS
The Athenæum Press
1902

Entered at Stationers' Hall

# PREFACE

"Electrical Problems" has been written in the belief that there is a field and should be a demand for this class of text-book.

In physical science generally, and especially in engineering, knowledge is of use only so far as we are able to calculate numerical results, and ability to obtain such results quickly and accurately is to be obtained only by extensive practice. Then, too, the ordinary mind arrives at a clear conception of general principles only by the way of concrete examples. To the average student mathematical formulæ are vague and uninviting until he has himself made practical application of them.

For some years the students of electrical engineering in Tufts College have been largely exercised in the solution of numerical problems, and both the experience of the instructors and the testimony of the students themselves clearly indicate the value of such work in rendering clear and precise their views of electrical phenomena. Especially valuable in clarifying the students' conceptions of physical relations are those problems whose answers appear in the form of curves, showing the effects of varying the quantities involved.

Most of the problems in this book have already been presented to the electrical classes in Tufts College. Nevertheless, among so large a number it is probable that errors, especially in the answers, remain to be corrected. Those communicating such corrections to them will receive the hearty thanks of

THE AUTHORS.

TUFTS COLLEGE,
September, 1902.

# CONTENTS

# ELECTRICAL PROBLEMS

## CHAPTER I

### THE FUNDAMENTAL UNITS

The system of units generally used in physical science employs the centimeter, the gram, and the second, respectively, as the units of length, mass, and time, and hence is known as the centimeter-gram-second, or more briefly as the C.G.S., system.

Since, however, commercial measurements are frequently made in terms of other units, as, for instance, feet, pounds, and minutes, it is important that one should be able to readily change from one system of units to the other.

It is assumed that the student is familiar with both the metric and the English systems. He should remember that

$$2.54 \text{ centimeters} = 1 \text{ inch};$$
$$453.6 \text{ grams} = 1 \text{ pound avoirdupois}.$$

### PROBLEMS

**1.** How many centimeters in a foot?

**2.** How many meters in a mile?

**3.** How many centimeters in a knot?

**4.** A No. 6 wire (Brown and Sharp Gauge, or B.S.G.) has a diameter of 162 mils. What is its diameter in centimeters?

NOTE. — The mil is the thousandth part of an inch.

5. A No. 10 wire, B.S.G., is .102 inch in diameter. What is its cross section in square mils and in circular mils?

NOTE.—A circular mil is the area of a circle of 1 mil diameter; a square mil is the area of a square of 1 mil side. The area in circular mils is thus the square of the diameter in mils, whereas the area in square mils is the square of the diameter in mils multiplied by $\frac{\pi}{4}$, or .7854. Thus the cross section of a wire of circular section .01 inch in diameter is 100 circular mils, or 78.54 square mils.

6. How many circular mils in the cross section of a wire 1.1683 centimeters in diameter?

7. What is the diameter in millimeters of a wire having a cross section of 1256.64 square mils?

8. How many cubic centimeters in a foot of a 500,000 circular mils wire?

9. What is the cross section in square millimeters of a 1,000,000 circular mils wire?

10. How many cubic centimeters in a mile of wire .1 inch in diameter?

11. How many grains in a gram?

12. How many grains in an ounce avoirdupois?

13. One kilogram per kilometer is how many pounds per 1000 feet?

14. One pound per 1000 feet is how many kilograms per kilometer?

15. A cubic centimeter of water weighs 1 gram. Find the weight in pounds of a cubic foot.

16. The specific gravity of drawn copper is 8.9. What is the weight of 1000 feet of a wire 102 mils in diameter?

17. How long a wire 1 millimeter in diameter can be drawn from 1 pound of copper?

18. How long a wire 1 mil in diameter can be drawn from 1 ounce of copper?

19. What is the weight in pounds per mile of a copper wire 0253 inch in diameter?

**20.** If 1000 feet of copper wire 460 mils in diameter weigh 641 pounds, what is the weight of 1 mile of copper wire 128 mils in diameter?

**21.** If a kilometer of copper wire .6438 millimeter in diameter weighs 2.89 kilograms, what is the weight in pounds of 1 mile of copper wire 11.3 mils in diameter?

# CHAPTER II

## CURRENT, ELECTRO-MOTIVE FORCE, AND RESISTANCE

The three electrical units most frequently employed are those of current, electro-motive force, and resistance.

By Ohm's law the resistance of a conductor through which a uniform current is flowing is expressed by the ratio of the electro-motive force to the current, whatever the system of units employed. In the practical system the unit of resistance is called the *ohm*, the unit of electro-motive force the *volt*, and the unit of current the *ampere*.

For purposes of computation Ohm's law may be expressed by any one of the following equations:

$$I = \frac{E}{R}, \quad R = \frac{E}{I}, \quad E = IR.$$

### PROBLEMS

**1.** What current is produced through a resistance of 5 ohms by an electro-motive force of 10 volts?

**2.** What current is produced through a resistance of 48 ohms by an electro-motive force of 12 volts?

**3.** Through what resistance will an electro-motive force of 15 volts produce a current of 3 amperes?

**4.** Through what resistance will an electro-motive force of 2.5 volts produce a current of 10 amperes?

**5.** What electro-motive force will produce a current of 4 amperes through a resistance of 4 ohms?

**6.** What electro-motive force will produce a current of .03 ampere through a resistance of 1000 ohms?

7. Through what resistance will 121 volts produce 11 amperes?

8. What electro-motive force will produce .65 ampere through 13 ohms resistance?

9. What current is produced by 10 volts acting through .25 ohm?

10. A dynamo generates 500 volts; the resistance of the circuit, including the dynamo, is 20 ohms. What is the current?

11. A 16 candle power lamp requires .5 ampere, and its resistance at full candle power is 200 ohms. What voltage must be impressed on its terminals?

12. What electro-motive force must a dynamo generate to supply an electroplating current of 20 amperes through a circuit whose total resistance is .2 ohm?

13. The electro-motive force supplying a current of 2 amperes is 40 volts. Determine the resistance of the circuit.

14. A dynamo generates an electro-motive force of 1150 volts and delivers a current of 20 amperes. The resistance of the circuit, including dynamo, is what?

15. A battery cell gives an electro-motive force of .9 volt, and has an internal resistance of 1.2 ohms. If its terminals are short circuited by a negligible resistance, what current will flow?

16. An electric bell has a resistance of 400 ohms and will not ring with a current of less than .03 ampere. Neglecting battery and line resistance, what is the smallest electro-motive force that will ring the bell?

17. At 20° C. No. 10 copper wire, B.S.G., has a resistance of .994 ohm per 1000 feet. An electro-motive force of 200 volts is impressed on a circuit of it 200 miles long. What current will flow?

18. What is the resistance per mile of No. 20 B.S.G. copper wire at 75° F. if a current of 5 amperes is passed through 100 feet of it and the reading of a voltmeter connected to the ends of the wire is 5.13 volts?

**19.** If a car heater is supplied with an electro-motive force of 500 volts from the trolley, how great must its resistance be that the current may not exceed 5 amperes?

**20.** A 5000 ohm galvanometer has its terminals connected to two points in a circuit between which there is a difference of potential of .01 volt. What current flows through the galvanometer?

**21.** What electro-motive force is needed for an incandescent lamp of 50 ohms resistance, through which flows a current of 1.04 amperes?

**22.** A battery whose electro-motive force is 100 volts is used to light a series of 4 lamps having a total resistance of $33\frac{1}{3}$ ohms. What current will flow?

**23.** If the magneto-generator for ringing a telephone bell gives an electro-motive force of 50 volts, what current will be transmitted through the circuit if the resistance of generator line and bell is 12,700 ohms?

**24.** What is the maximum possible current from a battery cell of 2.1 volts electro-motive force and .03 ohm internal resistance?

**25.** At 0° C. No. 9 B.S.G. copper wire has a resistance of .739 ohm per 1000 feet. A telephone trunk line of this wire is blown over on a trolley wire, short circuiting the trolley and the rail with a length of 30 feet. The telephone wire has a conductivity 98% that of pure copper. Supposing the potential difference between the trolley and rail to be maintained at 500 volts, what current will flow in the No. 9 wire?

# CHAPTER III

## RESISTIVITY

The resistivity, formerly called the specific resistance, of a substance is the resistance between opposite ends of a standard volume.

Physicists usually express resistivity in ohms per cubic centimeter at the temperature of 0° C. Thus the resistivity of a sample of copper in centimeter ohm measure is the resistance in ohms at 0° C. between opposite faces of a cube of this particular copper, each of whose edges is 1 centimeter long. Engineers frequently employ the circular mil-foot as a standard volume, in which case the resistivity is the resistance between opposite ends of a wire of circular section, 1 foot long and .001 inch in diameter.

Other standard volumes are used, as the meter-millimeter, the meter-gram, etc.; also resistivities are sometimes expressed in absolute units of resistance — billionths of an ohm — per cubic centimeter when that unit is employed as the standard volume. The latter unit of resistance might also be employed with the other standard volumes.

The resistance of any portion of a circuit uniform in material and of constant cross section equals the product of its length and resistivity divided by its area.

Generally an increase of temperature causes an increase of resistivity in metals.

In most pure metals the amount of this increase is about 1% for every 2.5° C. increase at and near 0° C.

Hence the temperature coefficients of pure metals are approximately .004.

## PROBLEMS

**1.** The resistance of 500 meters of copper wire with a cross section of .001 square centimeter is 79.5 ohms at 0° C. What is the resistivity of copper in centimeter ohm measure?

*Solution.*

$$\text{resistance} = \frac{\text{resistivity} \times \text{length}}{\text{area}}.$$

$$\text{resistivity} = \frac{\text{resistance} \times \text{area}}{\text{length}} = \frac{79.5 \times .001}{500 \times 100} = .00000159.$$

**2.** What is the resistivity in circular mil-foot ohm measure of copper?

**3.** At 60° C. what is the resistance of 1 mile of copper wire 42 mils in diameter?

**4.** At 77° F. how many feet of No. 10 copper wire, B.S.G., will have a resistance of 1 ohm? (See Appendix.)

**5.** If the temperature in Problem 4 is reduced to 32° F., by what per cent must the length of wire be increased to still have a resistance of 1 ohm?

**6.** A column of mercury 106.3 centimeters long, with a section 1 millimeter square, has a resistance of 1 ohm at 0° C. What is the resistance between opposite faces of an inch cube?

**7.** A mercury column is 4 millimeters long and 6 by 8 millimeters in section. What is its resistance at 0° C.?

**8.** A battery gives an electro-motive force of 208 volts and has a negligible resistance. The external circuit is a coil of 10,000 turns of copper wire 10 mils in diameter, the average length of a turn being 8 inches. What current will flow with the copper at a temperature of 0° C.?

**9.** If the copper wire of Problem 8 is heated to 100° C., by what per cent will the current be lessened?

**10.** At 0° C. 1037.3 centimeters of iron wire, with a section of 1 square millimeter, has a resistance of 1 ohm. What is the resistivity of iron in centimeter ohm measure?

**11.** What is the resistivity of iron in circular mil-foot ohm measure?

**12.** The tube of a mercury thermometer has a cross section of 1 square millimeter, the bulb being of such size that the column ascends 1 centimeter for 2° rise in temperature. The lowest reading is $-40°$ C. One terminal of a circuit is fixed at this point; the other moves so as to be always at the top of the column. If the temperature coefficient for the resistance of mercury is .00072 per degree C., what is the ratio of the resistances of the column at the temperatures of freezing and boiling water?

**13.** The resistance of a 1 inch cube of lead between opposite faces is .00000773 ohm. A roof of lead is 1000 feet long, 20 feet wide, and $\frac{1}{16}$ inch thick. At 0° C. what is its resistance between ends?

**14.** A lead cable sheath is 3 centimeters in outside diameter and 2 millimeters thick. Owing to impurities the conductivity of the sheath is 103% that of pure lead. If the temperature coefficient of resistance of lead is .004 per degree C., what will be the resistance of the sheath per kilometer at 25° C.?

**15.** The resistivity of platinum in centimeter ohm measure is .000009. At 0° C. what is the resistance of a platinum wire 1 millimeter in diameter, per meter of length?

**16.** Assuming a temperature coefficient for the resistance of platinum of .00247 per degree C., how long must a wire of No. 18 B.S.G. platinum wire be to have a resistance of 2 ohms at 60° C.?

**17.** At 35° C. what diameter platinum wire will have a resistance of .01 ohm per foot?

**18.** The resistivity of silver in centimeter ohm measure being .00000149, what diameter silver wire will have a resistance of 10 ohms per kilometer?

**19.** A circuit is composed of the following elements in series: 5 miles of No. 9 B.S.G. iron wire; a coil of 3000 turns of No. 30 B.S.G. copper wire, with an average length of $3\frac{1}{3}$ inches per turn; 6 mercury contacts, each 5 centimeters long, with a cross section of 1 square centimeter; 50 inches of No. 30 B.S.G. platinum wire. The copper has a conductivity

98% that of pure copper; the iron is impure, and at this temperature has a resistance 1.75 that of pure iron at 0° C.; the platinum and mercury are pure. The temperature of the circuit is taken at 77° F.; if an electro-motive force of 10 volts is impressed on the circuit, what current will be produced?

**20.** An aluminum wire 1 millimeter in diameter and 1 meter long has a resistance of .037 ohm at 0° C. What is the resistivity of aluminum in centimeter ohm measure?

**21.** What is the resistivity of aluminum in circular mil-foot ohm measure?

**22.** The temperature coefficient of resistance of aluminum is .00139. What is the resistance per mile of No. 3 B.S.G. aluminum wire, at 18° C.?

# CHAPTER IV

## ELECTRO-MOTIVE FORCES AND RESISTANCES IN SERIES GROUPING

When two or more sources of electro-motive force are arranged so that their electrical pressures act along a single circuit, the positive terminal of one being connected to the negative terminal of the next, either directly or through a section of the circuit, the positive terminal of the last and the negative terminal of the first being connected through an external circuit, the several sources of electro-motive force are said to be in series grouping.

A typical case is that of a zinc-carbon battery used to ring a bell. The zinc of one cell is connected to the carbon of the next, the zinc of the second cell to the carbon of the third, and so on, the free zinc and carbon terminals of the battery being connected to the bell terminals by the usual insulated wires.

In such a case the available electro-motive force is the algebraic sum of all the component electro-motive forces acting on the circuit. When these all act in the same direction the resultant is their arithmetical sum.

A circuit may be divided into several sections, the current passing to each one from the preceding. The resistances of the several sections are then said to be in series grouping. The total resistance of the several sections arranged in series is their arithmetical sum.

Unless otherwise stated, electro-motive forces acting on the same circuit are considered to act in the same direction, or accumulatively.

## PROBLEMS

**1.** Two electro-motive forces, one of 5 volts and the other of 27 volts, act in series on a circuit of two sections having resistances of 3 ohms and 5 ohms, respectively, arranged in series. What current will flow?

*Solution.* Total electro-motive force = 5 + 27 volts = 32 volts.
Total resistance = 3 + 5 ohms = 8 ohms.

$$I = \frac{E}{R} = 4 \text{ amperes.}$$

**2.** A battery is composed of 20 cells in series, each having an electro-motive force of 1.1 volts. Through what resistance, including that of the battery itself, will it produce a current of $\frac{2}{3}$ ampere?

**3.** A lighting circuit contains 40 lamps in series, each of $8\frac{1}{8}$ ohms resistance. If the resistance of the generator and line is 8 ohms, what electro-motive force must be maintained to supply a current of 3 amperes?

**4.** A lighting circuit 3 miles long is laid in an underground duct. Each conductor has an insulation resistance of 10 megohms. If the potential difference between the lines is 1000 volts, what is the leakage current?

**5.** Ten 200 ohm lamps are used in series in synchronizing two alternators. At a particular instant one generator gives an electro-motive force of 1414 volts in one direction and the other an electro-motive force of 614 volts in the opposite direction. What current flows through the lamps?

**6.** What is the current in the shunt field of a generator having a terminal voltage of 550 if the field resistance is 63 ohms and there is in series a rheostat with a resistance of 17 ohms?

**7.** A storage battery of 30 cells in series is being charged from a generator whose electro-motive force is 75 volts and whose resistance, including battery connections, is .6 ohm. The internal resistance of the battery is .03 ohm per cell and

its back electro-motive force is 2 volts per cell. What is the charging current?

**8.** During the process of charging the battery of Problem 7 the back electro-motive force rises to 2.1 volts per cell. What generator electro-motive force is needed to keep the charging current constant?

**9.** Keeping the electro-motive force of the generator of Problem 7 constant, what will be the charging current when the back electro-motive force has risen to 2.1 volts per cell?

**10.** A battery of 10 cells in series acts through an external circuit of 10 ohms resistance. On open circuit each cell has an electro-motive force of 1.5 volts and an internal resistance of 1 ohm. After a period of activity the effect of polarization is to increase the resistance .2 ohm per cell and to diminish the effective electro-motive force .4 volt per cell. At this stage, by what per cent is the current reduced from its value at the time of closing the circuit?

**11.** There are impressed on a circuit three electro-motive forces of 3, 4, and 5 volts, respectively. The sources of electro-motive forces have internal resistances as follows: that of 3 volts 4 ohms, that of 4 volts 5 ohms, and that of 5 volts 6 ohms. The resistance of the external circuit is 15 ohms. What is the current?

**12.** In Problem 11 what is the current if the 3 volts electro-motive force acts in opposition to the other two?

**13.** In Problem 11 what is the current if the 4 volts electro-motive force acts in opposition to the other two?

**14.** In Problem 11 what is the current if the 5 volts electro-motive force acts in opposition to the other two?

**15.** What is the current in Problem 11 if the source of 3 volts electro-motive force is removed?

**16.** What is the current in Problem 11 if the source of 4 volts electro-motive force is removed?

**17.** What is the current in Problem 11 if the source of 5 volts electro-motive force is removed?

**18.** What is the current in Problem 11 if the external resistance is removed?

**19.** The difference of potential between the carbons of an arc lamp is 40 volts. It is assumed that the arc has a back electro-motive force of 35 volts. If a current of 10 amperes flows, what is the resistance of the arc?

**20.** A motor driven from a battery of 60 volts potential takes a current of 5 amperes. If the resistance of the battery is 2 ohms and that of the motor and connections 1.5 ohms, what is the back electro-motive force of the motor?

**21.** A 3 phase dynamo has 4 collecting rings, A, B, C, D. One end of each coil is connected to a common ring and the other ends are connected, each to a separate ring. A current is passed successively from each ring to each of the others. An ampere-meter is placed in the circuit and a voltmeter connected to the rings between which the current passes with the following readings:

| *From* | *To* | *Volts* | *Amperes* |
|---|---|---|---|
| A | B | $\frac{16}{30}$ | 9.2 |
| A | C | $\frac{29}{30}$ | 8.7 |
| A | D | $\frac{29}{30}$ | 8.8 |
| B | C | $\frac{15}{30}$ | 9.3 |
| B | D | $\frac{15}{30}$ | 9.2 |
| C | D | $\frac{29}{30}$ | 8.7 |

Find the resistance between each pair of rings and state which ring is connected to the common junction.

**22.** Ten cells in series have each an internal resistance of 1 ohm and give an electro-motive force of 1.1 volts. There is an external copper circuit having, at 0° C., a resistance of $1\frac{2}{3}$ ohms. If the copper is placed in boiling water, what current will flow?

**23.** The same battery is used as in Problem 22. The external resistance at 50° C. is 10 ohms copper and 10 ohms iron. The temperature coefficient for the resistance of iron is .0054 per degree C. If the copper is cooled to 10° C. and the iron is heated to 90° C., what current will flow?

**24.** With the conditions the same as in Problem 23, except that the copper is heated to 90° C. and the iron cooled to 10° C., what current will flow?

**25.** With conditions the same as in Problem 23, except that the copper is heated to 80° C. and the iron to 210° C., what current will flow?

**26.** Suppose the heating in Problem 25 to have been caused by the flow of the current. In the meantime polarization has diminished the effective electro-motive force .3 volt per cell, and has increased the internal resistance .4 ohm per cell. Under these conditions, what would be the current in the circuit?

**27.** An electro-motive force of 100 volts is maintained at the terminals of a circuit which at 0° C. consists of 40 ohms copper and 60 ohms iron in series. Plot a curve of current as the temperature of the circuit rises to 350° C.

# CHAPTER V

## ELECTRO-MOTIVE FORCES AND RESISTANCES IN PARALLEL GROUPING

The ability of a conductor to transmit a uniform current without excessive heating varies inversely with its resistance or directly with the reciprocal of its resistance, its conductance. The idea of conductance is especially useful in dealing with multiple or branched circuits where the joint conductance of the parallel branches is the sum of the conductances of the individual branches. It follows, therefore, that the joint resistance of two or more conductors in multiple is the reciprocal of the sum of the reciprocals of their separate resistances. Hence the joint resistance of a branched circuit is less than the resistance on any single branch.

The law of Ohm may be expressed as follows:

$$I = EK, \; E = \frac{I}{K}, \; K = \frac{I}{E},$$

where $K$ is the conductance, the unit of which has been called the *mho*.

When two or more equal electro-motive forces are so arranged that they act all in the same direction along a given circuit, all the positive terminals being connected at one point of the circuit and all the negative terminals being joined at another point of the circuit, these common junctions being the terminals of the group, they are said to be connected in parallel or multiple grouping. The electro-motive force of the group as a whole is the same as that of any one of its individual elements.

## PROBLEMS

**1.** A divided circuit has two branches of 1 ohm and $\frac{1}{4}$ ohm, respectively. What is the joint conductance of the two branches? What is the joint resistance?

**2.** A divided circuit has resistances in the several branches of 1, 2, 4, 5, and 10 ohms, respectively. What is the conductance of the combination? What is the resistance?

**3.** To produce a current of 8 amperes, what electro-motive force must be impressed on a circuit containing the following elements in series: a coil of 6 ohms in multiple with one of 2 ohms, a line of 10 ohms, a group of 10 incandescent lamps in multiple, each of 200 ohms, a connecting wire of 4 ohms?

**4.** A circuit has two branches, one of 2 ohms with a current of 6 amperes, the other of 15 ohms. What current flows in the second branch and what difference of potential is maintained between the terminals of the circuit?

**5.** A circuit has three branches of 12, 3, and 6 ohms, respectively. If 4 amperes flow in the circuit containing 3 ohms, what current will flow in each of the others?

**6.** What is the resistance of a circuit of $A$ branches, each containing $B$ ohms?

**7.** Twenty cells, each of 2 volts electro-motive force and 1 ohm resistance, are connected in parallel to an external circuit of 1 ohm resistance. What current will flow in the external circuit?

**8.** A battery cell with an electro-motive force of 2 volts and an internal resistance of 2 ohms acts through an external circuit of 2 ohms resistance. By what per cent will the current be increased by putting another similar cell in multiple with the first?

**9.** In Problem 8 what would have been the percentage increase in the current by putting the second cell in series with the first?

**10.** How many cells like that of Problem 8 must be put in multiple to produce a current of 1 ampere in an external circuit of $1\frac{3}{4}$ ohms resistance?

**11.** A certain cell has an electro-motive force of 1.1 volts and a resistance of 1 ohm. Twenty such cells in series act through a circuit of 24 ohms. What is the current?

**12.** The battery of Problem 11 is arranged with the cells in 2 groups, each of 10 in series, the 2 groups being in multiple. What is the current?

**13.** The battery of Problem 11 is arranged with the cells in 4 groups, each of 5 in series, the 4 groups being in multiple. What is the current?

**14.** The battery of Problem 11 is arranged with the cells in 5 groups, each of 4 in series, the 5 groups being in multiple. What is the current?

**15.** The battery of Problem 11 is arranged with the cells in 10 groups, each of 2 in series, the 10 groups being in multiple. What is the current?

**16.** The battery of Problem 11 is arranged with all the cells in multiple. What is the current?

**17.** With an external resistance of 20 ohms, what is the greatest possible current with the battery of Problem 11, and by what arrangement of cells will it be produced?

**18.** With an external resistance of 5 ohms, what is the greatest possible current with the battery of Problem 11, and by what arrangement of cells will it be produced?

**19.** With an external resistance of 1.25 ohms, what is the greatest possible current with the battery of Problem 11, and by what arrangement of cells will it be produced?

**20.** With an external resistance of .8 ohm, what is the greatest possible current with the battery of Problem 11, and by what arrangement of cells will it be produced?

**21.** With an external resistance of .2 ohm, what is the greatest possible current with the battery of Problem 11, and by what arrangement of cells will it be produced?

**22.** With an external resistance of .05 ohm, what is the greatest possible current with the battery of Problem 11, and by what arrangement of cells will it be produced?

The student will note that in Problems 17–22 the arrangement of cells giving the maximum current is that in which the internal and external resistances are as nearly as possible equal. In general, with batteries or other sources of electro-motive force, when all the sources of electro-motive force are similar as to voltage and resistance, the arrangement for maximum current output is that which makes the ratio of the internal and external resistances as nearly as possible equal to unity.

**23.** Each of two dynamos has an electro-motive force of 1000 volts, and a resistance of 2 ohms between terminals. With an external resistance of 40 ohms, what current will be supplied with the dynamos in series, and what with them in parallel?

**24.** If the resistance between opposite faces of an inch cube of a battery solution is 10 ohms, what is the internal resistance of a battery having two plates, each of 10 square inches area, 1 inch apart?

**25.** A battery cell consists of 10 positive and 11 negative plates, 15 inches by 20 inches in size, arranged alternately positive and negative, and separated ½ inch. The battery solution has a resistivity of 300 in inch ohm measure. What is the internal resistance of the cell?

**26.** A lighting circuit 5 miles long is laid in an underground duct, each wire having an insulation resistance of four megohms per mile. If the potential difference between the wires is 2000 volts, what is the leakage current?

**27.** It is desired to shunt a galvanometer of 10 ohms resistance so that only 1% of the current shall pass through it. What must be the resistance of the shunt?

**28.** If in Problem 27 the remainder of the circuit has a resistance of 10 ohms, how and by what per cent will the main current be affected by the insertion of the above shunt? What will be the effect on the galvanometer current?

**29.** What resistance in series with the above-shunted galvanometer will compensate for the insertion of the shunt so as not to alter the main current?

**30.** Calculate a $\frac{1}{10}$, a $\frac{1}{100}$, and a $\frac{1}{1000}$ set of shunts for a 3000 ohm galvanometer.

**31.** A 5000 ohm galvanometer is connected to a thermoelectric pile of 3 ohms resistance. How and by what per cent will the insertion of a $\frac{1}{1000}$ galvanometer shunt affect the current in the galvanometer?

**32.** A galvanometer of 4000 ohms resistance is provided with a set of shunts and compensating resistances operated in the following manner: inserting the shunt plug in the first hole connects a $\frac{1}{1000}$ shunt, $s_1$, across the terminals of the galvanometer, and at the same time inserts a compensating resistance consisting of coils $r_1$, $r_2$, and $r_3$, connected in series; putting the plug in the second hole connects a $\frac{1}{100}$ shunt, $s_2$, across the galvanometer in series with $r_1$, $r_2$ in series with $r_3$ acting as the compensating resistance. When the plug is in the third hole a $\frac{1}{10}$ shunt, $s_3$, is connected across the galvanometer in series with $r_1$ and $r_2$, $r_3$ being the compensating resistance. Calculate the values of $s_1$, $s_2$, $s_3$, $r_1$, $r_2$, and $r_3$.

# CHAPTER VI

## DROP

The difference of potential or the drop in voltage, frequently termed simply *drop*, between any two points in a circuit in which a constant current is flowing, is numerically equal to the product of the current flowing in, and the resistance of, the conductor between the given points.

A little consideration will show that the sum of the separate drops in the various sections into which a circuit may be divided must equal the algebraic sum of all the electro-motive forces in the circuit; evidently, also, in the case of a divided circuit the drop is the same in each of several parallel branches.

### PROBLEMS

**1.** If a current of 1 ampere flows in a circuit, what is the drop in a section having a resistance of 1 ohm?

*Solution.* Drop $= IR = 1 \times 1 = 1$ volt.

**2.** A trolley wire of No. 0 B.S.G. has a resistance of .519 ohm per mile. What is the copper drop between the station and a car taking 20 amperes, 2 miles out on the line?

NOTE. — In the following problems trolley wire will be assumed No. 0.

**3.** A power station maintains a difference of potential of 550 volts between trolley and ground at the station. The resistance of the rail return is .04 ohm per mile. If there is only one car on the line, how far from the station must it be to have the pressure fall to 500 volts with a current of 35 amperes?

**4.** Seven cars a mile apart and each taking 15 amperes are on a trolley line 6 miles long. Beginning at the station, find the drop to each car, assuming .04 ohm per mile for the rail return.

NOTE. — This problem illustrates the importance of supplying trolley lines with large supplementary conductors, called *feeders*, and of frequently connecting these feeders to the trolley wire. By this means the resistance of the line is kept down and excessive drops prevented.

**5.** If the station in Problem 4 maintains a potential of 600 volts, what is the voltage at each of the seven cars?

**6.** A circuit consists of 8 ohms resistance in generator and line, and of 40 lamps in series, each of $8\frac{1}{3}$ ohms. With a current of 3 amperes, what will be the drop in the generator and line, and what the fall in voltage at each lamp?

**7.** An arc light dynamo of 30 ohms resistance supplies a current of 6.8 amperes through 15 miles of line wire of No. 6 B.S.G. to a series of 45 arcs, each adjusted to 47 volts. Find the electromotive force of the dynamo, the difference of potential at its terminals, and the drop on the line at 60° F. and at 0° F.

**8.** A lighting circuit consists of 10 groups of lamps in multiple between the line wires, the groups being 100 feet apart, and the nearest group 500 feet from the generator. Each group of lamps takes 5 amperes, and the resistance of the line is .102 ohm per 1000 feet. What is the drop from the generator to each group?

**9.** Instead of the arrangement described in Problem 8, one of the two line wires runs directly to the group of lamps farthest from the generator, and then back to the group nearest, connecting with each group on the return. All other conditions are as in Problem 8. What is the drop to each group?

**10.** Plot curves showing the conditions in Problems 8 and 9, using for abscissas the distances of the various groups from the generator, and for ordinates the voltages at the groups. Assume in each case that the voltage of the generator is 115 volts.

**11.** An electric railway 6 miles long is equipped with No. 0 trolley and 4 miles of feeder of No. 00, with a resistance of

.412 ohm per mile, connected to the trolley every half mile. At a certain instant of time 4 cars are running as follows: No. 1 is 1 mile from the station, taking 10 amperes; No. 2 is $2\frac{1}{2}$ miles from the station, taking 40 amperes; No. 3 is $4\frac{1}{2}$ miles from the station, taking 25 amperes; No. 4 is $5\frac{1}{2}$ miles from the station, taking 30 amperes. Neglecting the rail resistance, what is the voltage at each car if the station voltage is 550?

**12.** Assuming a 70 pound rail with a track resistance of .03 ohm per mile, what must be the station voltage in Problem 11 to give 500 volts at the last car?

**13.** An electric railway 5 miles long has two feeders; the first is No. 00 and extends 4 miles from the station, being tied to the trolley every half mile; the other is No. 0000, with a resistance of .259 ohm per mile, is 2 miles long, and is connected to the trolley at its farther end. The cars are situated as follows: No. 1 is starting from the station, taking 40 amperes; No. 2 is $1\frac{1}{2}$ miles out, taking 30 amperes; No. 3 is 3 miles out, taking 10 amperes; No. 4 is $4\frac{1}{2}$ miles out, taking 50 amperes. Neglecting the track resistance and assuming that the voltage at the car 3 miles out is 500, find the voltage at the station and at each of the other cars.

Suggestion. — In solving this problem we will first determine the distribution of current in the three conductors. This we may do by remembering that the drop in the 2 miles of No. 0000 feeder must be the same as that in the same length of trolley and No. 00 feeder. Hence call the current in the first $1\frac{1}{2}$ miles of trolley and No. 00 feeder $X$, and the current in the No. 0000 feeder $Y$; then $X + Y = 90$, and $1.5 \times .2297\, X + .5 \times .2297\, (X - 30) = 2 \times .259\, Y$.

**14.** Assuming the track resistance to be .025 ohm per mile, and keeping the station voltage in Problem 13 unaltered, what will be the voltage at each car?

**15.** A railway 7 miles long has a trolley, a No. 00 feeder 6 miles long tied to the trolley at every half mile, and a No. 0000 feeder 4 miles long connected to the overhead system at each mile; 7 cars a mile apart are on the line, the first being a mile

from the station, and each taking 25 amperes. Allowing .0212 ohm per mile for track return, what is the drop to each car?

**16.** A trolley road is arranged in the form of a square with sides 2 miles long, and the station in the middle of one side. From the station a No. 0000 feeder runs directly across the square to the middle of the opposite side, where it is connected to a No. 00 feeder extending .5 mile each way. From the station a No. 00 feeder extends 2.5 miles in each direction around the square. All No. 00 feeders are connected to the trolley at every half mile. A car at the middle of each side of the square takes 50 amperes, and one at each corner takes 20 amperes. Draw a diagram of the road and determine the current in each conductor and the voltage at each car, assuming 525 volts at the station, but neglecting the resistance of the rail return.

**17.** Allowing .0212 ohm per mile for the rail return in Problem 16, by what amount will the voltage at each car be diminished by the drop in the return circuit?

**18.** An electric road is in the form of a rectangle with sides of 2 and 4 miles, and the station is in the middle of one of the shorter sides. From the station a feeder with a resistance of .1 ohm per mile extends across the rectangle to the middle of the opposite side. Connecting the middles of the longer sides and the above-described feeder is a cross feeder of .2 ohm per mile. The combined resistance of trolley and feeder extending either way from the station to the middle of the longer sides is .2 ohm per mile, from these points to farther corners it is .3 ohm per mile, and from the corners to the middle of the side opposite the station it is .4 ohm per mile; 50 amperes are required at the middle of each side and at each corner of the rectangle, and 20 amperes at each point midway between. Determine the distribution of current and the drop to each of the above-described points, neglecting the resistance of the rail return.

**19.** Assuming 550 volts at the station and .02 ohm per mile for the resistance of the rail return, determine the voltage at each of the required points on the line, in Problem 18.

## CHAPTER VII

### POWER AND EFFICIENCY

The power, activity, or time rate of expenditure of energy in a circuit is numerically equal to the product of the current and electro-motive force in the circuit. This may be expressed by the equation $P = EI$.

Thus in practical units:

$$\text{watts} = \text{joules per second} = \text{volts} \times \text{amperes}.$$
$$1 \text{ horse power} = 746 \text{ watts}.$$

Furthermore the power absorbed in any portion of a circuit is equal to the product of the current and difference of potential between the extremities of that portion of the circuit, $p = eI$.

Since $e$, the difference of potential between two given points of a circuit, is equal to the product of the current and resistance between the given points, $e = Ir$, it follows that $p = I^2r$ for any portion of a circuit.

### PROBLEMS

**1.** A generator delivers a current of 100 amperes at a pressure of 100 volts. What power does it supply in kilowatts? How many horse power?

**2.** A current of 1.5 amperes flows through a circuit of 2 ohms resistance. How many watts are absorbed?

**3.** How many watts are expended in a 110 volt 16 candle power lamp requiring a current of .5 ampere? How many watts per candle?

**4.** The generator in Problem 1 has a shunt wound field with a resistance of 50 ohms. How many watts are absorbed in the field coils?

**5.** The armature of above generator has a resistance of .03 ohm. What power is lost in the armature conductors?

**6.** Assuming all other losses to be 1 kilowatt, how many horse power will be required to drive above generator at full load of 10 kilowatts?

**7.** An engine supplies 145 horse power to a generator delivering 181.82 amperes at an electro-motive force of 550 volts. What is the commercial efficiency of the generator?

*Solution.* 145 H.P. $= 145 \times 746 = 108{,}170$ watts, input.

$181.82 \times 550 = 100{,}000$ watts, output.

$$\frac{100{,}000}{108{,}170} = .9245 = 92.45\% \text{ efficiency.}$$

**8.** What is the commercial efficiency of the generator of Problems 1, 4, 5, and 6?

**9.** A battery is formed of 2 cells, each with 2 volts electromotive force and 1 ohm resistance. The cells are connected in series and supply current to a circuit of 2 ohms resistance. How much power is absorbed in the battery? how much in the external circuit, and what is the efficiency?

NOTE. — In the case of a battery the efficiency is the useful power, that is, the power absorbed in the external circuit, divided by the total power generated.

**10.** The cells of Problem 9 are connected in parallel. What power is absorbed in the battery? what in the external circuit, and what is the efficiency?

**11.** A battery has 24 cells like those of Problems 9 and 10. They may be connected in 8 different ways: all in series; with 12 pairs in series, each pair in parallel; with 8 groups of 3 in series, each group in parallel; with 6 groups of 4 in series, each group in parallel; with 4 groups of 6 in series, each group

in parallel; with 3 groups of 8 in series, each group in parallel; with 2 groups of 12 in series, each group in parallel; all in parallel. If the external circuit has 4 ohms resistance, what power is developed by the battery in each case?

**12.** Under the conditions of Problem 11, what power is available in the external circuit in each case?

**13.** Under the conditions of Problem 11, what is the efficiency of the system in each case?

**14.** Under the conditions of Problem 11, what per cent of the total resistance is in the external circuit in each case?

**15.** Plot a set of 5 curves for the above battery, using for abscissas the per cent of the total resistance included in the external circuit and for ordinates the current, the total power generated, the useful power, the power absorbed in the battery, and the efficiency.

**16.** A 100 kilowatt generator, giving a terminal electro-motive force of 125 volts, has an armature resistance of .005 ohm and a brush contact resistance of .00125 ohm on each side. The total loss by friction is 2100 watts, constant at all loads. The armature core loss is 1400 watts at 25% load, 1500 watts at 50% load, 1700 watts at 75% load, 2000 watts at full load, and 2400 watts at 125% load. The power absorbed by the field is 546, 590, 634, 678, and 722 watts at 25%, 50%, 75%, 100%, and 125% of full load, respectively. What is the total loss at each of the above percentages of full load?

**17.** What is the commercial efficiency of the generator of Problem 16 at each of the specified percentages of load?

**18.** A storage battery consists of 60 cells with an internal resistance of .003 ohm each. In charging, the cells are connected in 2 sets in multiple, each of 30 in series. The charging current is delivered by a generator having .15 ohm resistance, through a line of .02 ohm resistance. At the time of beginning the charge the battery gives a back electro-motive force of 1.95 volts per cell. What electro-motive force must be induced in the generator to give a charging current of 50 amperes?

**19.** Assuming the electrical efficiency of such a system as that of Problem 18 to be the percentage of the generated electro-motive force used to overcome the back electro-motive force of the battery, what is the electrical efficiency of the system of Problem 18?

**20.** If all the cells of the battery of Problem 18 are connected in series, what electro-motive force must be generated to keep the charging current 25 amperes per cell?

**21.** What is the electrical efficiency of the system of Problem 20?

**22.** When fully charged the cells of Problem 18 have a back electro-motive force of 2.2 volts each. If connected as in Problem 18, what electro-motive force must be generated at the end of the charging to keep the current up to its original value? What is the electrical efficiency of the system at this time?

**23.** When the battery is fully charged, what electro-motive force must be generated to keep the charging current at its original value with the connection of Problem 20?

**24.** An arc light dynamo generates a current of 9.6 amperes. The resistance of the machine is 19 ohms; there are 12 miles of No. 6 B.S.G. line wire at a temperature of 60° F. and 40 arcs, each taking 48 volts. What per cent of the total electrical power is used in the dynamo, what per cent on the line, and what per cent is utilized in the lamps?

**25.** What is the electrical efficiency of a 6.8 ampere arc dynamo maintaining a terminal potential difference of 2700 volts and having an internal resistance of 41.25 ohms?

**26.** Assuming an efficiency of 50% for the whole arrangement, what current will be used by a 250 volt electric hoist when raising 2500 pounds 200 feet per minute?

**27.** Assuming an average efficiency of 30%, what current will be required by a 500 volt railway motor to impart a velocity of 10 miles per hour in 20 seconds, to an 8 ton car on a level track?

**28.** Assuming that 70% of the electrical input is usefully expended, what current will be required by a 500 volt railway motor in propelling a 6 ton car up an 8% grade at a speed of 8 miles per hour?

**29.** A copper wire of 18.7 mils diameter is heated to redness by a current of 19 amperes, at which time there is a difference of potential of $4\frac{1}{2}$ volts between points on the wire 24.85 inches apart. How many horse power will be expended in heating a hollow copper cylinder 4 inches in diameter and 4 feet long to the same temperature?

# CHAPTER VIII

## THE MAGNETIC FIELD DUE TO A CURRENT

It is shown in electrical treatises that the magnetic force at a point, due to a current in a long rectilinear conductor, varies directly as the current and inversely as the distance of the point under consideration from the conductor.

Expressed in practical units, we have

$$H = \frac{.2\, I}{r},$$

where $H$ is the strength of the field in gausses, $I$ the current in amperes, and $r$ the perpendicular distance in centimeters of the point from the conductor. In a vacuum, and practically in air and other feebly magnetic media, a magnetic force $H$ produces a numerically equal flux density $B$, whose unit is also called a *gauss*. In magnetic media the magnetic flux per square centimeter, flux density, or magnetic induction $B$ is equal to the magnetic force $H$ multiplied by $\mu$, the magnetic permeability of the medium.

$$B = \mu H.$$

$\mu$ is unity for a vacuum, slightly greater than unity for air and most other substances, and possesses a considerable value only in iron, nickel, cobalt, and their alloys or compounds.

In general, then, we may say that $B = \frac{.2\, I}{r}$, but when the space about the conductor is occupied by iron

$$B = \frac{.2\, \mu I}{r}.$$

## PROBLEMS

**1.** What is the strength of the magnetic field 10 centimeters from a wire in which there is a current of 20 amperes?

**2.** What is the flux density 2.5 centimeters from a straight wire carrying a current of 6 amperes?

**3.** What is the field 3 feet from a railway feeder in which a current of 1000 amperes is flowing?

**4.** A point and 2 parallel conductors, 2 centimeters apart, are in a common plane, the point being 6 centimeters from the nearer of the two conductors. A current of 6 amperes flows in the nearer conductor, and an oppositely directed current of 4 amperes in the other. What is the resultant magnetic force at the given point?

NOTE. — At any point the direction of the magnetic force due to a current is perpendicular to the conductor and to the normal from the conductor to the given point. Moreover the directions of the current and of the resulting magnetic force have the same relation as the forward and rotational motions of a right-handed screw. Thus, if an observer looks along a current in the direction of flow, the magnetic force is concentric with the conductor and in the direction of motion of the hands of a watch.

**5.** Three parallel wires are each 1 foot from the other two; in two of the wires there is a current of 100 amperes, each in the same direction. What is the field in the space occupied by the third wire?

SUGGESTION. — The magnetic forces due to the two currents will differ 60° in direction. Their resultant will be found by the same process that would be employed in finding the resultant of two mechanical forces.

**6.** With the arrangement of Problem 5, what is the field at the third wire with the currents opposite in direction?

**7.** If each of the conductors of Problem 5 has a current of 50 amperes in the same direction, what is the field at a point equidistant from them?

**8.** In Problem 7 what is the field intensity at the equidistant point if 2 of the conductors have currents of 50 amperes in the same direction and the third the return current of 100 amperes?

It can be shown that the magnetic field at the center of a very short circular helix of $n$ convolutions and a radius of $r$ centimeters is $.2\,\pi n$ times the current in amperes divided by the radius.

$$H = \frac{.2\,\pi n I}{r}.$$

**9.** What is the magnetic force at the center of a circular convolution with a radius of 2 centimeters, carrying a current of 4 amperes?

**10.** A second convolution in the same plane as and concentric with that of Problem 9 has a diameter of 10 centimeters. How large a current must flow in this outer conductor to double the field at the center?

**11.** A third conductor in the same plane as and concentric with those of Problem 10 has a current flowing in the opposite direction to those of the other convolutions, and exactly neutralizing their effect. If the current in the third convolution is 6 amperes, what is its radius?

**12.** What is the field at the center of the coil of a tangent galvanometer of 25 convolutions and 20 centimeters mean radius when traversed by a current of .25 ampere?

**13.** What will be the deflection of the needle of the above galvanometer when placed with the plane of its coil in the magnetic meridian, at a place where the horizontal intensity of the earth's field is .175 gauss?

NOTE. — A field of 1 gauss exerts a force of 1 dyne on a unit pole. A field of strength $H$ exerts a force of $Hm$ dynes on a pole of strength $m$.

SUGGESTION. — In Fig. 1 let $CC_1$ represent the coil, $H_e$ the horizontal component of the earth's field, $H_g$ the magnetic field due to the current, $NS$ the needle, $l$ its length, $m$ the strength of its pole, and $\theta$ the angle of its deflection.

The moment of the couple due to the action of the current on the magnet is $H_g lm \cos\theta = \frac{.2\pi nI}{r} lm \cos\theta$ when $I$ is expressed in amperes. The moment of the couple due to the earth's field upon the magnet is $H_e lm \sin\theta$.

Since these are the only forces acting on the needle, they must balance if the deflection is to be constant; hence $\frac{.2\,\pi nI}{r} lm \cos\theta = H_e lm \sin\theta$, and $I = \frac{r}{.2\,\pi n} H_e \tan\theta$, or $\tan\theta = \frac{.2\,\pi nI}{rH_e}$. $\frac{r}{.2\,\pi n}$ is called the constant of the galvanometer.

**14.** What is the horizontal intensity of the earth's field when, under the conditions of Problem 12, the needle is deflected 55°?

**15.** A tangent galvanometer of 100 convolutions, the mean diameter of whose coil is 1 meter, has the plane of its coil placed in the magnetic meridian at a place where the strength of the earth's field is .18 gauss. What current will produce a deflection of 45°?

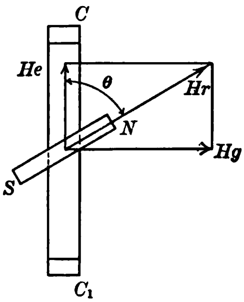



FIG. 1

**16.** The galvanometer of Problem 15 is arranged so that the whole instrument may be rotated about a vertical axis till the zero of its scale is again brought under the needle. With what current will it be necessary to rotate the instrument 30°? What will a galvanometer designed to be so used naturally be called?

**17.** What is the greatest current that the above instrument, used as in Problem 16, will measure?

It is shown in mathematical treatises on electricity that the magnetic field within and near the center of a long straight solenoid of a uniform number of turns per unit length is $4\,\pi$ times the current multiplied by the turns per unit of length. $H = \frac{.4\,\pi NI}{l}$, where $N$ is the whole number of convolutions of the solenoid, $l$ its length in centimeters, and $I$ the current in amperes.

**18.** Find the magnetic field near the center of a long helix of 4 convolutions per centimeter of length when traversed by a current of 15 amperes.

**19.** A solenoid 100 centimeters long and wound with three layers of wire of 540 convolutions each is traversed by a current of 6 amperes. What is the magnetic field near the center of the helix?

**20.** What will be the flux density $B$ at the center of a very long iron rod, whose permeability is 150, placed in the axis of the helix of Problem 19, the current being as before, 6 amperes?

**21.** What is the flux density within the wooden core of a helix of 513 convolutions per 30 centimeters of length, with a current of 5 amperes?

**22.** Find the magnetic force within a helix of 100 convolutions per foot of length, with a current of 20 amperes.

**23.** A circular solenoid is formed by winding wire upon a wooden ring whose diameter of section is 2 centimeters, the inside diameter of the ring being 10 centimeters and 275 convolutions being required to cover the ring. What is the magnetizing force within the wooden core per ampere of current?

**24.** What is the total magnetic flux per ampere within the solenoid?

**25.** If for the wooden core be substituted a wrought iron core with a permeability of 1500, with a current of 1 ampere, what will be the flux density? what the total flux?

**26.** What is the mean length of flux path in Problem 25?

Magnetic reluctance is the analogue of electrical resistance. The reluctance of the whole or of any part of a magnetic circuit varies directly as the length and inversely as the area of the section.

$$\text{reluctance} = \frac{\text{reluctivity} \times \text{length}}{\text{area}}.$$

In place of reluctivity its reciprocal, permeability, is generally used, so that reluctance $= \dfrac{l}{\mu A}$. The total reluctance of a

magnetic circuit formed of parts arranged in series is the sum of the reluctances of the component parts.

We have seen that the magnetizing force near the center of a long helix, or anywhere along the axis of a uniformly wound circular solenoid, is $.4\,\pi \frac{N}{l} I$. In the core of a circular solenoid, or in a coil, the product of this force into the length of the coil is called the *line integral of magnetic force*, or, more often, the *magneto-motive force* of the coil. Hence the magneto-motive force of a magnetizing coil is equal to $.4\,\pi$ times the ampere turns of the coil, $F$, in gilberts, equals $Hl$.

The total flux is the ratio of the magneto-motive force to magnetic reluctance. Flux, $\Phi$, in maxwells, equals $\dfrac{.4\,\pi NI}{\dfrac{l}{\mu A}}$, and when the magnetic circuit consists of parts, as in a dynamo, we have

$$\Phi = \frac{.4\,\pi NI}{\dfrac{l_1}{\mu_1 A_1} + \dfrac{l_2}{\mu_2 A_2} + \dfrac{l_3}{\mu_3 A_3} + \cdots}.$$

**27.** How long an air gap must be cut in the iron ring of Problem 25 to double the reluctance of the whole circuit?

**28.** If a certain part of the ring of Problem 25 consists of an alloy of iron having a permeability of 10, how long must the section of inferior permeability be to double the reluctance of the whole circuit?

**29.** With a magneto-motive force of 100 gilberts, what is the total flux along an air path 2 centimeters square and 50 centimeters long?

**30.** An anchor ring whose circular axis is 50 centimeters long is wound with 200 convolutions of wire carrying a current of 2 amperes. What is the magneto-motive force along the axis?

**31.** A circular solenoid has a core formed of a nickel ring. What will be the flux density per ampere turn per unit length, assuming a permeability of 100?

**32.** A complete magnetic circuit is divided into 5 sections, as follows: Section A, soft iron 8 centimeters long and $10 \times 30$ centimeters in section; Section B, soft iron 21 centimeters long and $15 \times 35$ centimeters in section; Section C, cast steel 30 centimeters long and with a circular section of 25 centimeters diameter; Section D, cast iron 50 centimeters long and $25 \times 40$ centimeters in section; Section E, air .75 centimeter long and $18 \times 40$ centimeters in section. What are the flux densities in the various sections, the total flux being 6,000,000 maxwells?

**33.** Under the conditions of Problem 32, the permeabilities of such grades of iron and steel as are now used in the construction of electrical machinery are likely to be about as follows: in $A, \mu = 63$; in $B, \mu = 1760$; in $C, \mu = 925$; in $D, \mu = 157$. How many gilberts will be required for each section?

**34.** Around Section C is wound a coil of 3750 convolutions. What current must flow in this coil to furnish the requisite magneto-motive force for the preceding problems?

**35.** With conditions as in Problem 34, what current would be required if the air gap were eliminated?

Faraday experimentally demonstrated that the whole quantity of electricity set in motion by the insertion or removal of $\Phi$ lines of magnetic induction in or from a coil of $N$ convolutions forming part of a circuit of $R$ units resistance is expressed by the equation $Q = \frac{\Phi N}{R}$. Clearly reversing the flux in a certain coil is equivalent to the insertion or removal of twice that flux. $\Phi N$ is called the total number of linkages of current and magnetic flux. The unit of quantity is the amount of electricity conveyed by unit current in unit time. The practical unit of quantity is the *ampere per second* or *coulomb*.

If a secondary coil of $N_2$ convolutions is placed inside a long helix of $N_1$ convolutions per unit length, the total change of induction in the secondary at each make or break of the primary current is expressed by the product of the flux density

inside the long helix and the integrated area of all the secondary convolutions.

$$\Phi N_2 = .4\,\pi N_1 \mu I \times N_2 A_2.$$

**36.** In Problems 18, 21, and 22 what are the total changes of induction in a secondary of 100 convolutions, each 3 centimeters in diameter, on breaking the primary current?

**37.** In each case of Problem 36 what quantity of electricity will be set in motion in a secondary circuit of 1000 ohms resistance by reversing the primary current?

SUGGESTION. — Since an ohm is $10^9$ absolute units of resistance and an ampere is $10^{-1}$ absolute units of current, when $R$ is taken in ohms and $I$ in amperes, $\Phi N_2$ must be multiplied by $10^{-8}$ to give $Q$ in coulombs.

**38.** On reversing the primary current in Problem 21, what quantity of electricity is set in motion in a secondary of 50 convolutions, 2 centimeters in diameter, with a total resistance of 5000 ohms in the secondary circuit?

**39.** An iron ring having a mean diameter of 20 centimeters and a cross section of 10 square centimeters has 100 turns of primary winding and 10 turns of secondary. Assuming a permeability of 1250 for the iron, what quantity of electricity is set in motion in the secondary circuit of 1500 ohms resistance in reversing a current of 5 amperes in the primary?

**40.** A ballistic galvanometer in the secondary circuit of Problem 38 gives a deflection of 150 scale divisions. What deflection will the same galvanometer give in the case of Problem 39? In each case the resistance of the galvanometer is included in the resistance given and the deflection of the galvanometer is assumed to be proportional to the quantity of electricity discharged through it.

# CHAPTER IX

## INDUCTANCE

The inductance or coefficient of self-induction of a coil is the integral, for all the convolutions, of the magnetic flux set up through each convolution per unit current in the coil.

Thus the inductance of a coil of 1000 convolutions through each of which 1,000,000 maxwells, or lines of magnetic flux, are set up by unit current or 10 amperes is, in absolute measure, 1,000,000,000, or, in the practical system of units, 1 henry. The henry is accordingly the inductance of a coil in which 100,000,000 linkages of current and magnetic flux are produced by each ampere of current.

It is frequently difficult and sometimes impossible to calculate the inductance of coils without iron cores, but with complete iron magnetic circuits so arranged that it may be assumed that all the flux lines pierce all the convolutions, the calculation becomes comparatively simple, if the permeability of the iron is known. Thus with a ring of soft iron of mean circumference $l$, area of Section A and permeability $\mu$, overwound with a layer of copper wire of $N$ turns through which there is a current of $I$ amperes, we have the magnetic flux $\Phi = \dfrac{.4\,\pi N I \mu A}{l}$.

$L$ is used as the symbol of inductance, and by definition

$$L\text{, in henrys, } = \frac{N\Phi \times 10^{-8}}{I};$$

$$\text{hence } L = \frac{.4\,\pi N^2 \mu A \times 10^{-8}}{l}.$$

The quotient of the inductance of a coil or circuit by its resistance is called its *time constant*, $T$. Physically the time

constant represents the time in seconds from the instant of closing the circuit required for the current to attain the $\frac{e-1}{e}$ part of its final value $\frac{E}{R}$, where $e = 2.718$ is the base of the Napierian system of logarithms. Thus in a circuit of 2 henrys inductance and 1 ohm resistance $T = \frac{L}{R}$, or 2; that is, 2 seconds after closing the circuit the current will be $\frac{1.718}{2.718} = .632$ of its final value.

It is shown in mathematical treatises on electricity that $i = \frac{E}{R}(1 - e^{-\frac{t}{T}})$, where $i$ is the value of the current $t$ seconds after closing the circuit.

### PROBLEMS

**1.** Find the inductance of a primary coil of 100 convolutions and secondary of 10 convolutions, wound on an iron ring of 20 centimeters mean diameter and 10 square centimeters section, the permeability of the iron being 700.

**2.** A transformer has an iron core of 292 square centimeters area and 47 centimeters in length. There are 220 turns in the primary and 22 turns in the secondary. Assuming the permeability of the iron to be 2000, what are the inductances of the two circuits?

**3.** If in Problem 2 the magnetic density had been raised to such a point as to halve the permeability of the iron, what would then have been the values of the primary and secondary inductances?

SUGGESTION. — It should be noted that while the inductance is constant in coils without iron, it is a variable quantity depending on the permeability, and therefore on the flux density, in coils with iron cores.

**4.** An iron ring has a mean diameter of 15 centimeters and a circular section with a radius of 1 centimeter. On it are wound 2 coils with 350 and 1000 convolutions, respectively.

If the permeability of the core is 650, what are the inductances of the two coils?

**5.** A transformer is wound with 50 turns in the primary coil and 1000 in the secondary. The magnetic circuit has a mean length of 50 inches and an area of 12 square inches. Assuming a permeability of 1800, what are the inductances of the two coils?

**6.** An iron ring has a mean diameter of 25 centimeters and a circular section of 1.5 centimeters radius. On it are wound 2 coils, a primary of 425 convolutions and a secondary of 17 convolutions. Find the inductance of each of the coils when the permeability of the iron core is 775.

**7.** A certain transformer has a magnetic circuit whose mean length is 35 inches, and whose gross section is 40 square inches, 10% of which is insulation. The primary coil contains 175 convolutions and the secondary 35 convolutions. If the permeability of the iron is 1250, what are the inductances of the two coils?

**8.** A reactance coil is formed of a ring of soft iron wire 17.5 square centimeters in section, and with a mean circumference of 50 centimeters, overwound with 650 convolutions of copper wire. Assuming the iron to have a permeability of 900, what is the self-induction of the coil?

**9.** Plot a curve showing the growth of current for 7 seconds after closing a circuit of 1 henry inductance and 1 ohm resistance, upon which is impressed an electro-motive force of 1 volt.

*Solution.* $$\frac{E}{R} = 1, \; i = \frac{E}{R}(1 - e^{-\frac{t}{T}}) = 1 - \frac{1}{e^t}.$$

When $t = .1, \frac{1}{e^t} = .9048, i = 1(1 - .9048) = .0952$, when $t = .2, i = .1813$, etc.

**10.** Plot a curve showing the rise of current in a circuit of 4 henrys inductance and 3 ohms resistance, upon which is impressed an electro-motive force of 10 volts.

**11.** If 1000 volts are impressed on a circuit of 10 henrys inductance and 4 ohms resistance, what will be the value of the

current at the end of $\frac{1}{120}$ second? what at the end of 1 second? what at the end of 5 seconds? what the final value?

**12.** A circuit contains .2 henry inductance and 5 ohms resistance. What is the value of the time constant and what will be the value of the current $T$ seconds after applying an electro-motive force of 60 volts?

**13.** A circuit contains 10 ohms resistance and has impressed on it an electro-motive force of 100 volts. Draw curves of current at the end of .5, 1, and 2 seconds as the inductance varies from 0 to 20 henrys.

**14.** A circuit of 50 ohms resistance has impressed upon it an electro-motive force of 100 volts. Plot curves showing the rise of current when the circuit contains inductances of 25 henrys, 50 henrys, and 100 henrys, respectively.

**15.** A circuit with 50 henrys inductance has impressed upon it an electro-motive force of 100 volts. Plot curves showing the rise of current, with resistances of 25 ohms, 50 ohms, and 100 ohms, respectively.

**16.** The instantaneous values, $i$, of the current through the field magnet coils of a dynamo were noted at various intervals of time, $t$, after closing the field magnet circuit, with the following results:

| $t$ in seconds | $i$ in amperes | $t$ in seconds | $i$ in amperes |
|---|---|---|---|
| .5 | 2.1 | 4 | 5.45 |
| 1.0 | 3.3 | 5 | 5.63 |
| 1.5 | 3.9 | 6 | 5.73 |
| 2.0 | 4.3 | 7 | 5.81 |
| 2.5 | 4.7 | 8 | 5.83 |
| 3.0 | 5.05 | 60 | 5.84 |
| 3.5 | 5.3 | | |

Plot the curve showing the rise of current in the field coils, and from it, by observing $T$, find the inductance, the resistance being 13.2 ohms.

If we neglect the falling off in the flux density near the ends, we may calculate the inductance of a long helix without an

iron core by the same formula used for a coil with a complete iron circuit. The field inside the long helix is expressed by

$$H = \frac{.4\,\pi NI}{l} \text{ and } \Phi = HA = \frac{.4\,\pi NAI}{l}; \text{ hence } L = \frac{.4\,\pi^2 r^2 N}{l} \times 10^{-9},$$

where $r$ is the radius of the helix.

For a circular coil of radius very large compared with its section, such, for instance, as the coil of a tangent galvanometer, the inductance is approximately expressed by

$$L = 4\,\pi N^2 r\left(\log_e \frac{16\,r}{a} - 2\right)10^{-9},$$

where $r$ is the radius of the coil and $a$ the side of the approximately square section.

When 2 coils are so situated that a current of 1 ampere in either sets up 100,000,000 linkages of magnetic flux and convolutions in the other, they are said to have a mutual inductance, $M$, of 1 henry. In the absence of iron mutual inductance as well as self inductance is constant.

The mutual inductance of 2 concentric helices of nearly the same radius will be approximately

$$M = \frac{4\,\pi^2 r^2 N_1 N_2 \times 10^{-9}}{l},$$

where $r$ is the radius of the inner coil and $N_1$ and $N_2$ are the numbers of convolutions in the 2 coils, respectively.

The value of $M$ for 2 short coaxial coils situated in the same plane and of nearly equal radii, $r$ and $r + c$, respectively, is approximately expressed by

$$4\,\pi N_1 N_2 r\left(\log_e \frac{8\,r}{c} - 2\right)10^{-9},$$

or if the plane of one of the coils is moved in the direction of its axis, so that the constant distance between the mean circular axes is $b$, then

$$M = 4\,\pi N_1 N_2 r\left(\log_e \frac{8\,r}{b} - 2\right)10^{-9}.$$

**17.** What is the inductance of a coil of 100 convolutions wound on a glass rod of 2 centimeters radius, the winding occupying a length of 25 centimeters?

**18.** A solenoid with 200 convolutions in a length of 30 centimeters is wound on a wooden cylinder having a radius of 3 centimeters. What is its inductance?

**19.** What is the inductance of a solenoid of 5000 convolutions in a length of 13 feet, wound on a cylinder of wood 1 foot in diameter?

**20.** What is the mutual inductance of two concentric coils 65 centimeters long, of 200 and 630 convolutions, respectively, the inner coil being wound directly on a paper tube 16 centimeters in diameter?

**21.** What is the inductance of the coil of a tangent galvanometer consisting of 100 convolutions of a mean radius of 1 meter, wound in a groove 1.5 centimeters square?

**22.** Two circular coils are wound in parallel grooves 1 centimeter apart. Each coil has a mean radius of 25.75 centimeters, a section 1.3 centimeters square, and contains 234 convolutions. What is their mutual inductance?

**23.** What is the inductance of the 2 coils of Problem 22 when joined in series?

SUGGESTION. — The inductance of 2 coils in series is the sum of their separate inductances plus twice their mutual inductance.

**24.** A groove with a rectangular section 1 centimeter wide and 2 centimeters deep is turned in a wooden cylinder, the radius of the bottom of the groove being 20 centimeters. In this groove is wound a primary coil of 25 convolutions, having a depth of 1 centimeter and a secondary coil of 400 convolutions also having a depth of 1 centimeter. What is their mutual inductance? What is the inductance of the 2 coils joined in series?

A very important calculation is that of the inductance of transmission lines. The formula for the inductance, per unit

length, of straight parallel conductors carrying equal but oppositely directed currents is

$$L = 2\left(2\,\mu \log_e \frac{d}{r} + \frac{\mu'}{2}\right),$$

in which $\mu$ is the permeability of the medium, $\mu'$ that of the wires, $d$ the distance between the axes of the wires, and $r$ the radius of the wires.

In case of non-magnetic wires hung in air, the inductance in henrys for the length $l$ is expressed by the equation

$$L = l\left(9.21 \log_{10} \frac{d}{r} + 1\right)10^{-9}.$$

**25.** What is the inductance per kilometer of a line of 2 copper wires 45 centimeters apart and 1 centimeter in diameter?

**26.** What is the inductance of a line 200 miles long consisting of 2 wires 15 inches apart and .1616 inch in diameter?

**27.** What is the inductance per mile of 2 No. 4 B.S.G. copper wires strung 18 inches apart?

# CHAPTER X

## THE CONDENSER

The capacity of a condenser varies inversely with the thickness of the dielectric and directly with the area of the conducting surfaces and the permittivity, or dielectric constant, formerly known as the specific inductive capacity, of the insulating material. Expressed in C.G.S. electrostatic units,

$$C = K\frac{A}{4\pi d};$$

$C$ being the capacity in absolute units, $K$ the permittivity, $d$ the thickness of the dielectric, and $A$ the effective area of the conducting plates, usually planes, cylinders, or spheres. The quantity of electricity required to produce a difference of potential $E$ between the plates of a condenser of capacity $C$ is $Q = EC$; whence $C = \frac{Q}{E}$ and $E = \frac{Q}{C}$.

The commonly used unit of capacity is the microfarad, or millionth part of a farad, the practical unit of a farad being the capacity that will be charged to a potential of 1 volt by a quantity of 1 coulomb, or by a current of 1 ampere flowing for a second into the condenser.

The above relations between $Q$, $E$, and $C$ are true for both the absolute and the practical units. The equation

$$C = K\frac{A}{4\pi d}$$

becomes, when $C$ is the capacity in microfarads and $A$ and $d$ are expressed in square and linear centimeters, respectively,

$$C = K\frac{A}{4\pi d \times 9 \times 10^5}.$$

The following table gives approximate values of the permittivity of various dielectrics in use in condensers:

| | |
|---|---|
| Air or vacuum | 1 |
| Paraffined paper | 2 |
| India rubber, pure | 2.34 |
| India rubber, vulcanized | 2.94 |
| Gutta-percha | 4.2 |
| Mica | 5 |
| Glass | 6 |

## PROBLEMS

**1.** What charge will be imparted to a condenser having a capacity of 100 units by a potential difference of 1000 units?

**2.** How many volts are required to impart a charge of .001 coulomb to a capacity of 4 microfarads?

**3.** What capacity will hold a charge of 900 units under a difference of potential of 45?

**4.** What quantity will charge a capacity of .01 microfarad to a potential of 6 volts?

**5.** How long must a current of 2.5 amperes flow to charge a capacity of .21 microfarad to a potential of .6 volt?

**6.** What is the capacity of a condenser formed of 2 plates 1 meter square, separated by 1 millimeter of air?

**7.** Of how many plates 1 meter square, separated by 1.3 millimeters of air, and with the 2 external plates of the same polarity, must a condenser be composed to have a capacity not less than .4 microfarad?

**8.** What must be the area of the plates in Problem 7 if the condenser is to have a capacity of precisely .4 microfarad?

**9.** If the condenser of Problem 8 have its plates separated by mica .5 millimeter thick, what is its capacity?

**10.** What is the capacity of a condenser built of 200 sheets of tin foil, each 100 square centimeters in effective area, separated by paraffined paper .1 millimeter thick?

**11.** A condenser is to be built of sheets of tin foil having an effective area 8 centimeters square, separated by .5 millimeter of pure rubber. What number of plates will give the nearest approach to .01 microfarad?

**12.** What will be the exact capacity of the condenser of Problem 11?

**13.** Keeping the tin foil sheets of the condenser of the previous problem 8 centimeters wide, what must be their effective length to produce a capacity exactly .01 microfarad?

**14.** A condenser is built of 160 circular sheets of tin foil separated by mica .5 millimeter thick. How many tin foil surfaces will be of one polarity? What must be the diameter of the sheets that the condenser may have a capacity of $\frac{1}{3}$ microfarad?

Two concentric, cylindrical, metallic surfaces insulated from each other may serve as a condenser. The equation for capacity becomes

$$C = K \frac{l}{2 \log_e \frac{R}{r}}$$

in absolute electrostatic units, where $l$ is the length of the metallic cylinders, $R$ the internal radius of the outer, and $r$ the external radius of the inner. In microfarads

$$C = K \frac{2.413\, l}{10^7 \log_{10} \frac{R}{r}},$$

$l$, $R$, and $r$ being expressed in centimeters as in the formula for the capacity in absolute units.

**15.** What is the capacity in microfarads per mile of a copper wire .4 inch in diameter, contained within and separated from a hollow metallic cylinder by .06 inch of vulcanized rubber?

**16.** The outer tube of Problem 15 is .1 inch thick and is covered with .06 inch of gutta-percha. If the whole is immersed in water, what is the capacity, in microfarads per mile, of the condenser formed by the outer tube and the water?

**17.** A wire 4 millimeters in diameter and insulated with 5 millimeters of gutta-percha is placed in water. What is the capacity in microfarads per kilometer?

It is shown in mathematical treatises on electricity that the charge $q$ in a condenser of capacity $C$, $t$ seconds after being connected with a source of constant potential $E$, is expressed by

$$q = EC\,(1 - e^{-\frac{t}{RC}}),\ \text{or}\ q = Q\left(1 - \frac{1}{e^{\frac{t}{RC}}}\right),$$

where $Q$ is the final charge, $R$ the resistance in circuit with the condenser, and $e = 2.718$ is the base of the Napierian system of logarithms. $RC$ is called the time constant of the circuit and is the time in seconds from the instant of closing the circuit to the instant when the charge reaches the $\frac{e-1}{e} = .632$ part of its final value. This expression is true expressed in either absolute or practical units.

**18.** Draw a curve showing the amount of charge at each instant of time in a condenser of 1 microfarad capacity, charged through a resistance of 400 ohms by an electro-motive force of 100 volts.

*Solution.* $R = 400$, $C = 10^{-6}$, $RC = .0004$ second.

$$Q = EC = 100 \times 10^{-6} = .0001 \text{ coulomb},\ q = .0001\left(1 - \frac{1}{e^{\frac{t}{.0004}}}\right).$$

When $t = .0001,\ \frac{t}{RC} = .25;\ q = .0001\,(1 - .7788) = .00002212.$

When $t = .0002,\ q = .00003934$, etc.

It is further shown that where $i$ denotes the instantaneous value of the current flowing into a condenser, $i = \frac{Q}{RC}$ times $e^{-\frac{t}{RC}}$.

**19.** Draw the curve showing the instantaneous values of the charging current in Problem 18.

**20.** In what multiple of $RC$ will a condenser receive its full charge to within one part in 1000?

*Solution.* $q = Q - Qe^{-\frac{t}{RC}},\ \frac{Q-q}{Q} = e^{-\frac{t}{RC}} = \frac{1}{1000},$

$t = RC \log_e 1000 = 6.908\ RC$ seconds.

**21.** In what multiple of $RC$ will a condenser receive its full charge to within one part in 1,000,000?

**22.** A condenser of 4 microfarads capacity is charged through a resistance of 1000 ohms by an electro-motive force of 500 volts. Calculate the instantaneous value of the charge at the end of .001 second, .006 second, and .02 second.

**23.** Calculate the instantaneous values of the current flowing into the condenser, under the conditions of and at the instants specified in Problem 22.

**24.** A condenser of .5 microfarad capacity is charged through a resistance of 500 ohms. What is the time constant $T$? With 75 volts impressed upon the circuit, what will be the charge and what the current $T$ seconds after closing the circuit?

**25.** A condenser of 2 microfarads capacity is being charged by an electro-motive force of 250 volts. Plot curves showing charge at the end of .0001 second, .0005 second, and .002 second as the resistance varies from 0 to 500 ohms.

**26.** Plot curves of current under the conditions of and at the instants specified in Problem 25.

**27.** A circuit containing a resistance of 200 ohms has impressed upon it an electro-motive force of 100 volts. Plot curves showing the instantaneous values of the charge as the capacity varies from 0 to 5 microfarads, at the end of .0001 second, .0005 second, and .002 second.

**28.** Plot curves showing the values of the current under the conditions of and at the instants specified in Problem 27.

The capacity of an aërial line $l$ centimeters long, consisting of 2 wires separated by a space of $d$ centimeters and each $r$ centimeters in radius, is, in microfarads, $C = \dfrac{l}{8 \log_{10} \frac{d}{r} \times 10^6}$.

**29.** What is the capacity per kilometer of a line of 2 copper wires 5 millimeters in diameter, separated by 50 centimeters?

**30.** What is the capacity per mile of 2 No. 0 B.S.G. wires hung 2 feet apart?

**31.** What is the capacity of 100 kilometers of transmission line, the 2 wires being 1 meter apart and each 8 millimeters in diameter?

**32.** What is the capacity of a line 150 miles long, consisting of 2 No. 00 B.S.G. wires hung 5 feet apart?

The joint capacity of condensers in multiple is the sum of their separate capacities. The joint capacity of condensers in series is the reciprocal of the sum of the reciprocals of their separate capacities.

**33.** What is the joint capacity of 3 condensers of 2, 3, and 4 microfarads, respectively, all joined in multiple?

**34.** What is the joint capacity of the above condensers joined in series?

**35.** Two condensers of 2 and 3 microfarads, respectively, are joined in multiple. In series with this arrangement is a condenser of 4 microfarads capacity. What is the capacity of the combination?

# CHAPTER XI

## THERMOELECTRICITY

In a circuit formed of two wires of different metals, one of whose junctions is maintained at a higher temperature than the other, there is a resultant electro-motive force set up around the circuit. Electro-motive forces have been observed in nearly all metals but lead, when heated unequally in parts, and accordingly lead has been selected as the standard with which to compare other metals in expressing the thermoelectric power of couples, or the resultant electro-motive force around the circuit for a difference in temperature of 1° C. of the two junctions of the couple. This thermoelectric power is not affected by the presence of any conductor used to join the ends of the members of the couple, such as solder or a galvanometer, and is dependent on the mean temperatures of the junctions.

The following table from Gerard's *Electricity and Magnetism* (page 182) enables one to calculate the thermoelectric power of several metals taken with lead by the formula

$$e = \left(a + b\frac{t + t'}{2}\right)(t - t'),$$

where $e$ is the resultant electro-motive force in volts around the circuit, $a$ and $b$ physical constants of the metal used, and $t$ and $t'$ the temperatures of the hotter and cooler junctions, respectively, in degrees Centigrade.

| *Metal* | *a* | *b* |
|---|---|---|
| Copper | −.00000134 | −.0000000094 |
| Iron | −.00001715 | +.0000000482 |
| German silver | +.00001194 | +.0000000506 |

A positive value of $e$ shows a tendency of the current to flow to lead across the warmer junction and a negative value of $e$ a resultant electro-motive force in the opposite direction. When two metals other than lead are used in the couple the values of $a$ and $b$ are the differences between the above values of $a$ and $b$ for the two metals.

## PROBLEMS.

1. A circuit is formed of iron and lead, the junctions being kept at $-5°$ C. and $+5°$ C., respectively. What is the resultant electro-motive force around the circuit, and in which direction does the current flow?

NOTE. — Unless otherwise stated, the current direction will be assumed to be that across the warmer junction.

*Solution.* For iron, $a = -.00001715$, $b = +.0000000482$.

$$e = \left(-.00001715 + .0000000482\,\frac{5 + (-5)}{2}\right)(5 - (-5)) = -.0001715 \text{ volt.}$$

The current leaves the lead.

2. If each of the junctions of Problem 1 has its temperature raised 10°, what is the direction and value of the resultant electro-motive force?

3. A circuit contains a piece of iron and a piece of copper wire soldered together. If the temperatures of the junctions are, respectively, 120° C. and 20° C., what is the value and direction of the resultant electro-motive force?

4. If the temperatures of the junctions in Problem 3 are changed to 420° C. and 0° C., what is the value and direction of the resultant electro-motive force?

5. A circuit is formed of lead and German silver with one junction kept at 0° C. Plot a curve showing the electro-motive force in the circuit as the temperature of the other junction changes from 0° C. to 250° C.

6. Plot a curve showing the variation of the resultant electro-motive force in a copper-iron circuit with one junction kept in

ice water and the temperature of the other varying from 0° C. to 600° C.

The thermoelectric couple is frequently used for the measurement of high temperatures, and for this purpose it is found that only platinum and its alloys of iridium or rhodium are practicable. Le Chatelier and Boudouard give the following table of the electro-motive force in microvolts set up in junctions of a platinum wire with one of platinum alloyed with 10% of iridium or rhodium, at certain temperatures.

| *Temperature* (Degrees C.) | *Iridium* | *Rhodium* |
|---|---|---|
| 100 | 517 | 565 |
| 448 | 3,228 | 3,450 |
| 930 | 11,000 | 8,500 |
| 1,500 | | 15,100 |

The net electro-motive force in the circuit being the difference between those of the two junctions, if this can be observed and the temperature of the cooler obtained, the temperature of the hotter can be deduced from a curve, giving the electro-motive force set up at each temperature throughout the necessary range.

This net electro-motive force can be obtained in two ways: first, by balancing it against the drop in a certain section of a known resistance, $R$, when traversed by the current from a standard cell. If $E$ is the electro-motive force of the cell, $I$ its current through the known resistance $R$, and $r$ the adjustable resistance in which the drop balances $e$, we have $e = Ir$, $E = IR$, and $e = E\frac{r}{R}$. Second, the electro-motive force $e$ may be found by connecting the ends of the couple to the terminals of a galvanometer whose resistance is constant and so large that the slightly variable resistance of the couple, due to change in temperature, may be neglected. We have, then, $e = IR$, where $R$ is the resistance of the circuit and $I$ the current set up in it by the electro-motive force $e$.

**7.** Construct, from the table given above, curves of the variation with temperature of the electro-motive force at a junction of a platinum and a platinum with 10% iridium wire and at a junction of a platinum and a platinum with 10% rhodium wire.

**8.** A junction of platinum and platinum iridium 10% alloy is used to determine the temperature of a potter's furnace. It is connected to the terminals of a galvanometer of 5000 ohms resistance whose temperature is 18° C. What current will flow if the temperature of the furnace is 650° C.?

**9.** If in Problem 8 the deflection is 11.5 millimeters of scale, what temperature of the furnace is indicated by a deflection of 18 millimeters?

**10.** If the above couple is heated to 915° C. and the terminals are connected to a coil of copper of 2250 ohms resistance when kept in ice water, what current will flow?

**11.** A thermoelectric couple is formed of a wire of platinum and one of platinum and rhodium in the proportion of 9 to 1. The junction is heated to 1375° C. and the terminals in a room at 20° C. are connected to a galvanometer. A standard cell giving an electro-motive force of 1.432 volts at this temperature sets up a current of .01 ampere in a wire of 5 ohms resistance per meter. The terminals of above galvanometer are connected to points on this wire so that the difference of potential thereby produced between the terminals of the galvanometer opposes that due to the couple. For no deflection, how far apart must the contacts be?

**12.** Later it is found necessary to separate the contacts 18 and then 7 centimeters. What temperatures of the hotter junction are then indicated?

## CHAPTER XII

### ELECTRO-CHEMISTRY

When a current of electricity is passed through a chemical compound in the liquid state the liquid is in general decomposed; the more electro-positive products of the decomposition, metals, etc., being deposited upon the negative terminal or electrode, and the more electro-negative products, oxygen, etc., upon the positive electrode. This decomposition is known as *electrolysis*, the liquid is called an *electrolyte*, and the products of the electrolysis are termed *ions*.

The following laws were discovered by Faraday:

1. The amount of an ion deposited and of the electrolyte decomposed is proportional to the amount of electricity that has traversed the electrolyte.

2. The amount of electrolytic action is the same in each of several electrolytic cells arranged in series.

3. The amount of an ion set free is equal to the total quantity of electricity that has passed through the electrolyte multiplied by the electro-chemical equivalent of the ion.

The following are the electro-chemical equivalents of some common elements in grams per coulomb of electricity:

| | |
|---|---|
| Hydrogen | 0.0000104 |
| Gold | 0.00068 |
| Silver | 0.00112 |
| Copper (from cupric salt) | 0.000328 |
| Nickel | 0.000304 |
| Zinc | 0.000337 |
| Oxygen | 0.0000829 |
| Chlorine | 0.000367 |

## PROBLEMS

1. A current of 4 amperes flows through water. At what rate are hydrogen and oxygen liberated?

*Solution.* Four amperes are 4 coulombs a second. Four coulombs a second, liberate $4 \times .0000104 = .0000416$ gram of hydrogen and $4 \times .0000829 = .0003316$ gram of oxygen per second.

2. If the current of Problem 1 is passed through hydrochloric acid, what elements will be liberated, and at what rate?

3. From a solution of copper sulphate how much copper will be deposited per hour by a current of 1 ampere?

4. Two electrolytic baths are arranged in series in a circuit, one containing copper sulphate and the other silver nitrate. If 40 grams of copper are deposited hourly, what is the current, and how much silver is deposited in 10 minutes?

5. A piece of metal weighing 400 grams is to be plated with 5% of its weight of gold. It is placed in a solution of gold chloride, and a current of 10 amperes is used. How long must it remain in the bath?

6. A casting is to have deposited on it 2 pounds copper and then 2 pounds nickel. It is to be left in the copper bath for 10 hours. With the same current how long must it remain in the nickel bath?

7. A current of 20 amperes is divided between two branches, one containing an electrolytic gold-plating bath, the other a vessel of water to be decomposed. The current divides so that equal weights of oxygen and gold are separated. What are the currents, and how much hydrogen and oxygen are separated?

8. In a plant for electrolytically refining copper a current of 1000 amperes flows 23 hours per day, 300 days per year, through 60 cells arranged in series. How many tons of copper are refined each year?

9. A bath for the electric deposition of copper runs 8 hours per day, with a current of 500 amperes. How long must it run to deposit enough copper for 1 mile of No. 00 B.S.G. wire?

10. A copper voltameter, consisting of 5 cells in multiple, is connected in series with an ammeter, a variable resistance, and a storage battery. A constant current is maintained for exactly 1 hour, when the gain in grams in the weight of the cathodes was found to be, respectively, 1.536, 1.473, 1.621, 1.574, and 1.512. What should have been the ammeter reading?

11. A constant current deposits 3.628 grams of silver in 45 minutes. What is the value of the current?

# CHAPTER XIII

## ALTERNATING ELECTRO-MOTIVE FORCES AND CURRENTS

Alternating or periodic electro-motive forces and currents may be represented by clock diagrams, in which the instantaneous value of the periodic function is represented by the projection of a uniformly rotating vector upon the diameter of a circle, or by wave diagrams in which the instantaneous value of the function is represented by an ordinate progressing with uniform velocity along the axis of abscissas, whose length at any instant is the above-described projection.

A review of the methods of generating electro-motive forces and currents will show that the positive and negative half waves must generally be similar in shape. Frequently these curves approximate to sine curves, and being periodic they may, by Fourier's theorem, be considered as the resultant of two or more sine waves.

Even where they differ markedly from a simple sine curve there is always a simple sine wave of the same periodicity, called the equivalent sine wave, that may for most purposes be substituted for the actual periodic curve.

Since electrical energy varies as the square of the electro-motive force, or as the square of the current, alternating electro-motive forces and currents are measured by the square root of the mean of the squares of the equidistant ordinates of the particular wave diagrams. This $\sqrt{\text{mean}^2}$ (square root of the mean square) value is called the virtual value of the electro-motive force or current as the case may be, and all alternating current ammeters and voltmeters are made to indicate virtual values.

The equation for a harmonically varying electro-motive force is $e = E \sin \omega t$, where $e$ is the instantaneous value of the electro-motive force, $E$ the maximum value; $\omega$, numerically equal to $2\pi$ times the periodicity, is the linear velocity of the ordinate of the sine curve, or the angular velocity of the vector in the clock diagram, and $t$ is the time in seconds from the beginning of the reckoning to that instant at which the electro-motive force has the value $e$.

The equation for a periodic but non-harmonic electro-motive force will vary from the above only in containing more than one term on the right-hand side. Nearly all the electro-motive forces met with in practice may be very approximately expressed by the following equation:

$$e = E_1 \sin \omega t + E_3 \sin (3\,\omega t - \theta_3) + E_5 \sin (5\,\omega t - \theta_5),$$

in which $\theta_3$ and $\theta_5$ represent angles by which the zero of the particular harmonic is displaced from the zero of the fundamental.

The fact that the positive and negative half waves are usually similar in shape shows that commonly only the odd numbered harmonics, $3\,\omega$, $5\,\omega$, etc., occur in practice. The higher harmonics $E_7 \sin (7\omega t - \theta_7) + E_9 \sin (9\,\omega t - \theta_9)$, etc., when they occur, are usually too small to materially affect the shape of the resultant wave.

## PROBLEMS

1. Draw the clock and sine wave diagrams of an electro-motive force whose maximum value is 10 volts, produced by a rectangular coil turning in a uniform magnetic field at the rate of 10 revolutions per second.

2. What is the virtual value of the electro-motive force in Problem 1?

Suggestion. — The $\sqrt{\text{mean}^2}$ value of the ordinate of a sine curve is $\frac{1}{\sqrt{2}}$ of the maximum ordinate.

**3.** Add to the clock and sine wave diagrams of Problem 1 a vector and sine wave representing a current with a maximum value of 5 amperes lagging 30° in phase behind the electro-motive force.

**4.** What is the virtual value of the current in Problem 3?

**5.** Draw the clock and sine wave diagrams of a current of 21.21 amperes lagging 50° behind an electro-motive force of 35.35 volts.

NOTE. — In speaking of volts and amperes, unless otherwise specified, virtual values of current and electro-motive force are always given.

**6.** Plot the curve represented by $e = 100 \sin 503\, t$.

**7.** Combine with the curve of Problem 6 the curve represented by $e = 40 \sin\,(503\, t - 30°)$.

SUGGESTION. — Evidently the zero of this curve will be displaced $\frac{1}{12}$ of a period to the right of the other curve. The 2 curves are combined by adding their contemporaneous ordinates.

QUERY. — What can you say concerning the resultant of 2 sine curves of the same periodicity?

**8.** Plot the curve represented by

$$e = 120 \sin 80\, \pi t + 50 \sin 400\, \pi t,$$

which means that of the 2 components 1 has a maximum of 120 volts and a periodicity of 40, the other a maximum of 50 volts and a periodicity of 200.

**9.** Plot the curve represented by

$$i = 50 \sin 200\, \pi t + 15 \sin\,(600\, \pi t - 135°).$$

**10.** Plot the curve represented by

$$e = 50 \sin 100\, \pi t + 10 \sin\,(500\, \pi t - 90°).$$

**11.** Plot the curve represented by

$$i = 100 \sin 100\, \pi t + 15 \sin 300\, \pi t + 10 \sin\,(500\, \pi t - 180°).$$

**12.** Draw the clock and sine wave diagrams of the two electro-motive forces of a 2 phase alternating current armature, the maximum value of the electro-motive forces being 100 volts, and the periodicity 60.

SUGGESTION. — The rotating vectors $OA$ and $OB$ (Fig. 2) may represent the maximum values of the electro-motive forces in the 2 armature circuits, $Oa$ being the instantaneous value of the electro-motive force in one circuit at a certain instant, and $Ob$ the contemporaneous value of the electro-motive force in the other circuit. The sine waves may be drawn separately, each with reference to its particular vector.

13. Draw the clock and sine wave diagrams of the 3 electro-motive forces of a 3 phase armature with star grouping of the

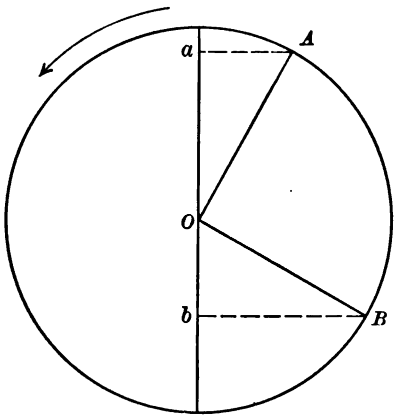



FIG. 2

coils, the maximum values of the electro-motive forces being 1000, and the periodicity 25. Also draw the wave diagrams of the electro-motive forces between the lines.

When the wave form $A$ (Fig. 3) of a periodic function is not a simple sine curve the virtual values are less easily obtained, but the following method enables them to be obtained graphically.

Rotate a vector about a point, laying off at definite intervals the value of the ordinate of the wave diagram at an angular advance from the beginning of the cycle equal to the angle through which the vector has rotated. If, as is usually the case, the positive and negative half waves are equivalent, only half a cycle need be laid off, since this half will give a closed

curve as shown in $A$ (Fig. 4), which is the polar diagram of the wave $A$ (Fig. 3). With a planimeter determine the area of the

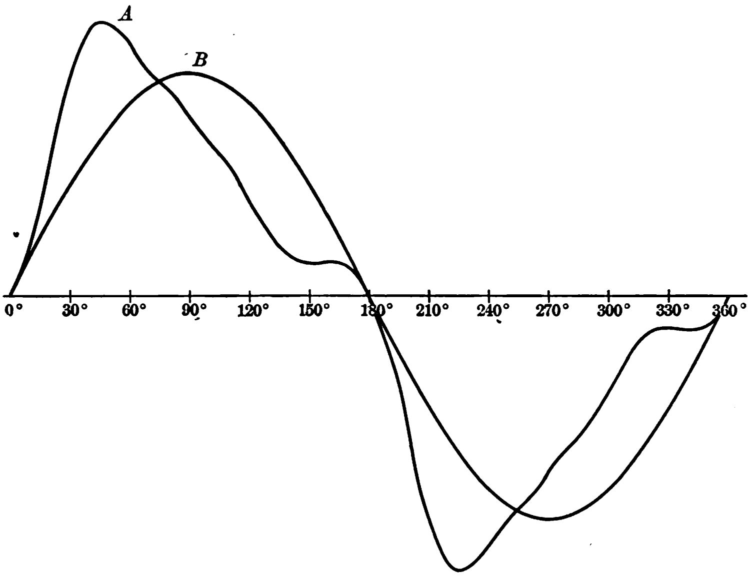



FIG. 3

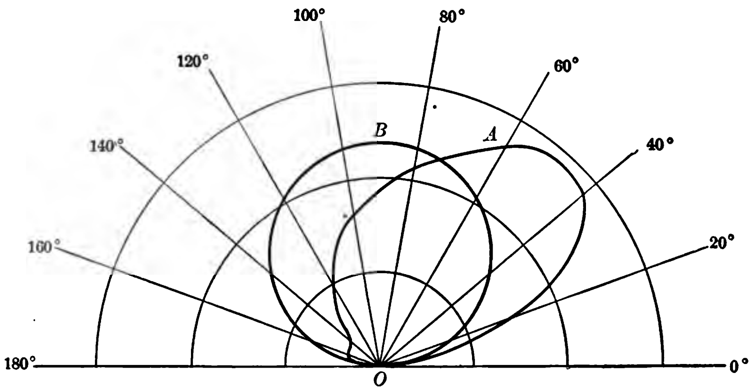



FIG. 4

curve and construct a circle $B$ of equal area, tangent to the horizontal axis at the origin. This circle is the polar diagram

of the sine curve having the same $\sqrt{\text{mean}^2}$ value of the equidistant ordinates as the original curve. Hence the diameter of this circle is the maximum ordinate of what is known as the equivalent sine curve, *B* (Fig. 3).

**14.** A certain electro-motive force is the resultant of 2 simple harmonic waves; the first has a maximum value of 100 volts and a periodicity of 50; the second a maximum value of 15 volts and a periodicity of 150, and agrees with the first in phase at the beginning of each period of the first. Draw the resultant wave form, determine the virtual value, and draw the equivalent sine wave.

**15.** A certain current wave is the resultant of 2 simple sine waves; the fundamental has a maximum value of 100 amperes and a periodicity of 50; the harmonic a maximum value of 30 amperes and a periodicity of 150, and lags in phase 90° behind the fundamental at the beginning of each period. Draw the resultant wave form and determine the virtual value of the current.

# CHAPTER XIV

## COMBINATION OF ALTERNATING ELECTRO-MOTIVE FORCES

Two harmonically varying electro-motive forces of the same periodicity in series may be added exactly as two mechanical forces are added, *i.e.*, by a vector or clock diagram.

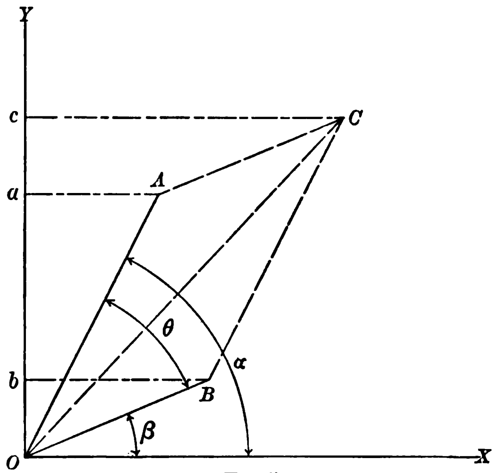



FIG. 5

In Fig. 5 let $OA$ and $OB$ represent the maximum values of $E_1$ and $E_2$, the two electro-motive forces to be added, $\theta$ the angle of phase difference between $E_1$ and $E_2$.

In the diagram $e_1 = E_1 \sin \omega t$, represented by $Oa$, is the instantaneous value of the first electro-motive force, and $e_2 = E_2 \sin (\omega t - \theta)$, represented by $Ob$, is the contemporaneous value of the second electro-motive force.

The vector sum of $OA$ and $OB$ is evidently $OC$, and the instantaneous value of $OC$ is $Oc$. Now whatever the position

of the parallelogram $AOBC$ when rotated about the center $O$, the sum of the projections $Oa$ and $Ob$ must always be equal to $Oc$. Evidently, then, the sum of the two harmonically varying electro-motive forces of the same periodicity is itself a harmonically varying electro-motive force of the same periodicity as its components and intermediate in phase between them.

From the trigonometry of the figure we have

$$\overline{OC}^2 = \overline{OA}^2 + \overline{OB}^2 + 2\ OA \times OB \cos\theta$$

and

$$\cos\theta = \frac{\overline{OC}^2 - \overline{OA}^2 - \overline{OB}^2}{2 \times OA \times OB}.$$

We have already seen that with harmonically varying quantities virtual values have a fixed relation to maximum. The vector diagram therefore is as applicable to the composition of virtual as of maximum values.

### PROBLEMS

**1.** Two electro-motive forces of the same periodicity and of 80 and 60 volts, respectively, differing in phase by 37°, act in series. What is the resultant voltage?

**2.** The electro-motive forces of Problem 1 differ in phase by 30°. Find their resultant.

**3.** The electro-motive forces of Problem 1 differ in phase by 60°. Find their resultant.

**4.** Two electro-motive forces of the same periodicity and each of 100 volts differ in phase successively by 30°, 60°, 90°, 120°, 150°, and 180°. Find the resultant voltage in each case.

**5.** Two electro-motive forces of the same periodicity and of 40 and 30 volts, respectively, are in series. Their resultant is 50 volts. What is their phase difference?

**6.** If the 40 volt component of Problem 5 leads, by what angle does the resultant lag behind it?

**7.** If the 2 electro-motive forces of Problem 5 have a resultant of 60 volts, by what angle do they differ in phase? By what angle does the 40 volt component lead the resultant?

**8.** Two equal period electro-motive forces of 100 and 25 volts, respectively, are in quadrature. What is the resultant?

**9.** Two electro-motive forces are in quadrature. One is 1000 volts and their resultant is 1010 volts. What is the second component and what is its phase relation to the resultant?

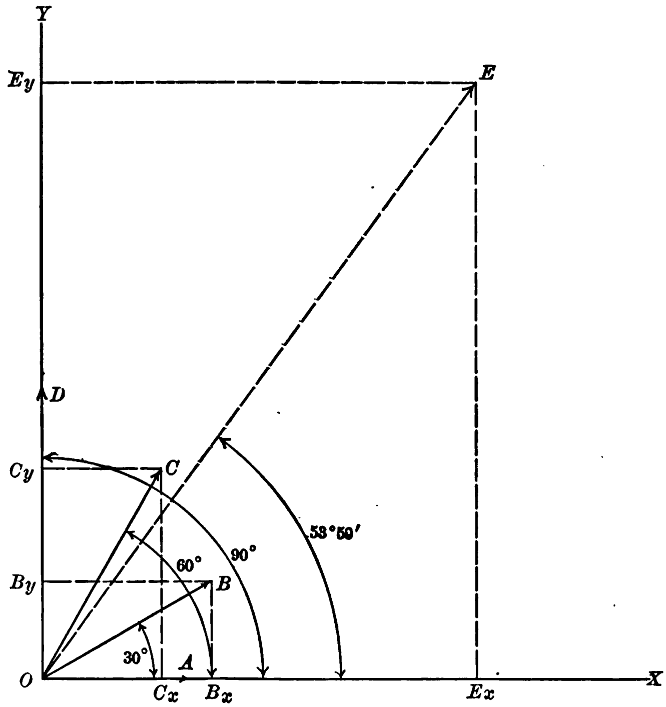



FIG. 6

**10.** Four equal period electro-motive forces of 30, 40, 50, and 60 volts, respectively, are in series; they successively differ in phase by 30°. Find the resultant in magnitude and phase.

SUGGESTION. — The 4 above electro-motive forces may be combined in pairs by the vector diagram, and the 2 resultants thus obtained combined for the final resultant. Where there are more than 2 components, however, it is usually preferable to combine them by the method of rectilinear coördinates; that is, resolve each electro-motive force into 2 components along

perpendicular axes; add all the $X$ components thus obtained and all the $Y$ components; their sums are the $X$ and $Y$ components of the resultant electro-motive force. Thus in Fig. 6 we may lay out the 30 volt component $OA$ on the $X$ axis and the 60 volt component $OD$ on the $Y$ axis, with the 40 volt and the 50 volt components $OB$ and $OC$ between at the proper angles; then denoting by $OA_x$ and $OA_y$ the $X$ and $Y$ components of $OA$, etc., we have

$$\begin{array}{llll}
OA_x = 30 \cos 0^\circ = 30 & & OA_y = 30 \sin 0^\circ = 0 \\
OB_x = 40 \cos 30^\circ = 34.64 & & OB_y = 40 \sin 30^\circ = 20 \\
OC_x = 50 \cos 60^\circ = 25 & & OC_y = 50 \sin 60^\circ = 43.3 \\
OD_x = 60 \cos 90^\circ = \underline{0} & & OD_y = 60 \sin 90^\circ = \underline{60} \\
OE_x = \quad 89.64 & & OE_y = \quad 123.3
\end{array}$$

The resultant $OE$ of $OE_x$ and $OE_y$ in quadrature is 152.4. This is therefore the resultant of all 4 original components. $OE$ is in advance of $OA$ by $\tan^{-1}\dfrac{123.3}{89.64} = 53^\circ\ 59'$.

**11.** The same electro-motive forces as in Problem 10 have phase angles of 0°, 60°, 120°, and 180°, respectively. Find the value and phase angle of the resultant.

**12.** Three electro-motive forces, each of 100 volts, are in series with phase angles of 30°, 150°, and 270°. What is their resultant in magnitude?

**13.** A 3 phase star connected armature generates 1000 volts in each armature circuit. The phase angles of the 3 electro-

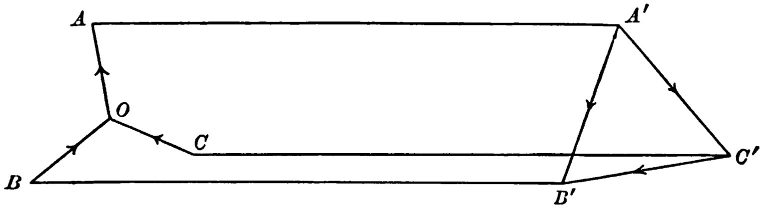



FIG. 7

motive forces being respectively 0°, 120°, and 240°, what will be the magnitude and phase angles of the electro-motive forces between the line wires?

SUGGESTION. — In Fig. 7, $OA$, $OB$, and $OC$ are the vectors representing the electro-motive forces in the 3 armature circuits. $AA'$, $BB'$, and $CC'$ are the 3 line wires, and $A'B'$, $B'C'$, and $C'A'$ are the vectors representing the 3 line voltages.

**14.** Power is transmitted over 3 wires from a 1000 volt 2 phase alternator, 2 of its 4 armature leads being connected at the brushes. Calling the phase angles of the 2 armature electro-motive forces 0° and 90°, what in magnitude and phase are the 3 line voltages?

**15.** Power is transmitted over 4 wires from a 4 phase alternator made by connecting the 2 armature circuits of a 1000 volt 2 phase alternator at their middle points. What are the magnitudes of the 4 line voltages?

# CHAPTER XV

## COMBINATION OF ALTERNATING CURRENTS

Two alternating currents uniting in a common conductor are combined precisely as two alternating electro-motive forces in series.

Thus in Fig. 8 let there be a harmonic current of 30 amperes in $A$ and a harmonic current of 40 amperes in $B$, the two currents differing 90° in phase. Then, by the vector diagram (Fig. 9), there will be a current of 50 amperes in $C$; for, since the point of junction $O$ is without capacity, it is evident that the instantaneous value of $C$ is the sum of the instantaneous values of $A$ and $B$.

Accordingly, as in Fig. 10, $C$ may be plotted as the algebraic sum of the contemporaneous ordinates of the sine curves $A$ and $B$, differing in phase by 90°. In the problems of this chapter sine waves of current are assumed unless otherwise stated.

### PROBLEMS

**1.** In a quarter phase system of 2 equal currents what is the ratio of each outgoing current to the current in the common return?

NOTE. — A quarter phase system has two currents with a phase difference of 90°, *i.e.*, in quadrature.

**2.** What is the resultant of 2 currents, each of 40 amperes, differing 120° in phase?

**3.** In a 3 phase system with 3 equal currents, differing in phase by 120°, what is the value of the current in the common return?

NOTE. — It thus appears that in a balanced symmetrical 3 phase system a fourth conductor serves no purpose whatever.

**4.** An unbalanced quarter phase system has currents of 12 amperes and 5 amperes, respectively, in its branches. What is the magnitude and phase angle of the current in the common return?

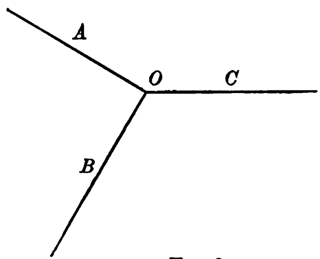



FIG. 8

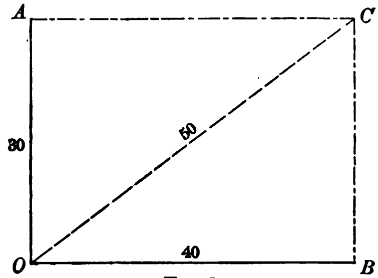



FIG. 9

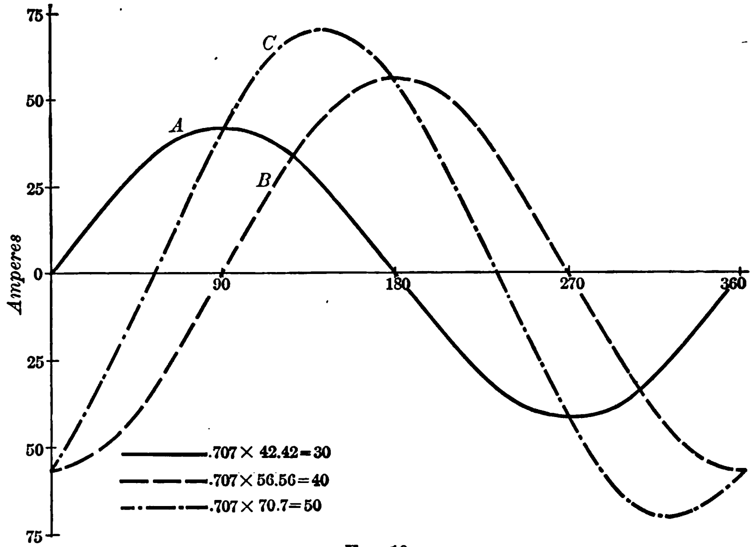



FIG. 10

**5.** An unbalanced 3 phase system has currents in the 3 legs of 4, 5, and 7 amperes, respectively, differing in phase by 120°. What is the current in a fourth conductor serving as a common return?

SUGGESTION. — Solve by the method of rectangular coördinates, as described in Chapter XIV, page 66.

**6.** A current divides between 2 branches; after division it is found that one branch carries 10 amperes, the other 6 amperes, and the phase difference between the two currents is 30°. What was the current before division in magnitude and phase?

SUGGESTION. — This may be solved graphically, as shown in Fig. 11. Draw $OA$ and $OB$ making angle $AOB = 30°$; lay off $OC$ and $OD$ in the ratio of 6 : 10. Complete the parallelogram $OCED$ with $OC$ and $OD$ as adjacent sides, and draw the diagonal $OE$. $OE$ will represent the current before division in magnitude and phase on the scale of $OA$ and $OB$. Angle $EOD$ or $EOC$, the phase angle between the original current and a component, may be found with a protractor or by trigonometry.

**7.** Two generators in parallel supply a current of 25 amperes to a line. The currents in the 2 armatures are respectively

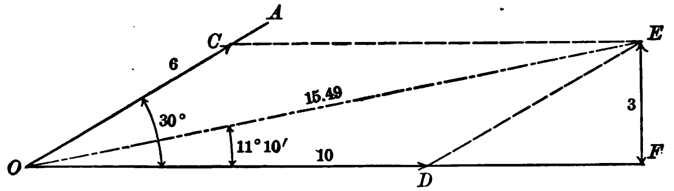



FIG. 11

10° ahead of and 25° behind the main current in phase. What is the magnitude of the current in each armature?

**8.** Two currents of 10 amperes each, differing 160° in phase, unite in a common return. Find the magnitude and phase of the return current.

**9.** Currents of 5, 6, 7, and 8 amperes, having phase angles of 0°, 30°, 90°, and 150°, respectively, unite in a common conductor. What is the resultant current in magnitude and phase?

**10.** Currents of 10, 20, 30, 40, 50, 60, 70, and 80 amperes, with phase angles of 0°, 45°, 90°, 135°, 180°, 225°, 270°, and 315°, respectively, unite in a common conductor. What is the resultant current in magnitude and phase?

**11.** Currents of 20, 35, 10, 90, 3, 12, 47, 20, and 30 amperes have phase angles of 30°, 135°, 330°, 60°, 180°, 210°, 90°, 300°,

and 45°, respectively. They unite in a common conductor. What is the resultant current in magnitude and phase?

**12.** A symmetrical 3 phase armature with delta grouping of armature circuits generates 100 amperes in each circuit. What is the current in each of the line wires?

**13.** A condenser and kicking coil are placed in multiple in an alternating current circuit. The condenser has a current of 1 ampere 85° in advance of the main current, and the kicking coil a current of 1 ampere 85° behind the main current in phase. What is the value of the main current?

# CHAPTER XVI

## IMPEDANCE

The electro-motive force required to establish an alternating current through a simple resistance, or the ohmic drop in that resistance, is, as is the case with direct currents, the product of the current and resistance, $E = IR$.

All circuits, however, possess more or less inductance and capacity, both of which, when the circuit is traversed by an alternating current, give rise to reactive electro-motive forces in quadrature with the current. The electro-motive force due to inductance is 90° behind the current in phase, and is numerically

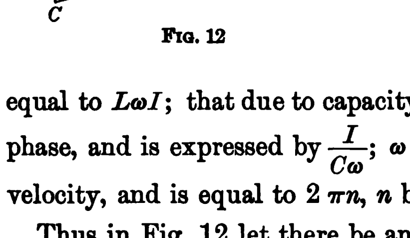



FIG. 12

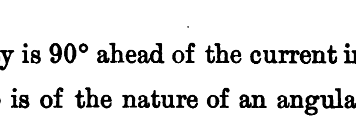



FIG. 13

equal to $L\omega I$; that due to capacity is 90° ahead of the current in phase, and is expressed by $\frac{I}{C\omega}$; $\omega$ is of the nature of an angular velocity, and is equal to $2\pi n$, $n$ being the periodicity.

Thus in Fig. 12 let there be an alternating current of virtual value $I$ and periodicity $n$ in a circuit of resistance $R$ and inductance $L$. Then $OB = RI$ represents the effective electro-motive force, or that required to overcome the resistance, and $OC = L\omega I$ the reactive electro-motive force due to inductance,

$90^\circ$ behind $RI$ in phase. To overcome $OC$ there must be impressed on the circuit an equal and opposite electro-motive force $OD$, which is accordingly $90^\circ$ ahead of $OB$. The total impressed electro-motive force $E$ must therefore be the resultant or vector sum, $OA$, of $OB$ and $OD$, equal to $I\sqrt{R^2 + L^2\omega^2}$.

If we divide each side of the triangle of electro-motive forces by the current $I$, we shall have the ohmic triangle $OAB$ (Fig. 13), in which the base represents the resistance $R$, the perpendicular the reactance $L\omega$, and the hypoteneuse the impedance $\sqrt{R^2 + L^2\omega^2}$, all expressible in ohms.

It is evident that the current will lag behind the electromotive force by an angle $\theta$, and that

$$\tan\theta = \frac{L\omega}{R},\ \sin\theta = \frac{L\omega}{\sqrt{R^2 + L^2\omega^2}},\ \cos\theta = \frac{R}{\sqrt{R^2 + L^2\omega^2}},$$

and that the current

$$I = \frac{E}{\sqrt{R^2 + L^2\omega^2}}.\quad i = \frac{E}{\sqrt{R^2 + L^2\omega^2}}\sin(\omega t - \theta),$$

where $i$ is the instantaneous value of the current, and $\omega t$ the contemporaneous phase angle of the electro-motive force.

## PROBLEMS

**1.** What is the impedance in a circuit of 4 ohms resistance and 3 ohms reactance?

*Solution.* Impedance $= \sqrt{R^2 + L\omega^2} = \sqrt{4^2 + 3^2} = 5$ ohms.

**2.** What is the inductance in Problem 1 if the frequency is 159?

**3.** By what angle does the current lag behind the electromotive force in the circuit of Problem 1?

**4.** Assuming the virtual value of the electro-motive force in Problem 1 to be 100 volts, draw the sine curves of the various electro-motive forces and current.

5. With a frequency of 50, what is the impedance of a coil of 5 ohms resistance and .01 henry inductance?

6. With an impressed electro-motive force of 100 volts and a frequency of 60 cycles per second, what current will be produced in a circuit of 4 ohms resistance and .02 henry inductance?

7. A current of 18.48 amperes, with a periodicity of 33, flows in a coil of 10 ohms resistance and .02 henry inductance. What is the impressed electro-motive force?

8. What must be the periodicity in Problem 7 to give a current of 8 amperes?

9. By what angles will the current lag behind the electromotive force in Problems 7 and 8?

10. With a periodicity of 25 cycles per second, the impedance of a circuit of 12 ohms resistance is 20 ohms. What is the inductance?

11. With 25 cycles per second, the impedance of a circuit of .05 henry inductance is 20 ohms. What is the resistance?

12. Find analytically and graphically the impedance of a circuit of 200 ohms resistance and .4 henry inductance, with a periodicity of 70.

SUGGESTION. — Draw ohmic triangle as in Fig. 13 and measure the hypotenuse.

13. Find graphically the reactance of a circuit whose resistance is 4 ohms and whose impedance is 10 ohms.

14. Two No. 1 B.S.G. wires 18 inches apart extend as feeders from an electric light station to a center of distribution 3 miles away. Neglecting capacity, what is the impedance of the feeders at a temperature of 60° F. and a periodicity of 125?

15. A single phase transmission line 40 miles long consists of 2 No. 0 B.S.G. wires 30 inches apart. Neglecting capacity, what is the impedance of the line at the temperature of 0° C., with a periodicity of 25?

16. A circuit consists of 2 sections in series, one of 12 ohms resistance and .015 henry inductance, the other of 10 ohms

resistance and .05 henry inductance. Find graphically the impedance at a frequency of 40.

SUGGESTION. — $2\pi n = \omega = 251.3$. Draw $OA$ (Fig. 14) = 12, $AB$ perpendicular to it = $.015\,\omega$. Then $OB = \sqrt{12^2 + (.015\,\omega)^2}$ = impedance of the first section. Draw $BC$ parallel to $OE = 10$, $CD$ perpendicular to it = $.05\,\omega$. Then $BD = \sqrt{10^2 + (.05\,\omega)^2}$ = impedance of the second section. Draw $OD = \sqrt{(10+12)^2 + (.015 + .05)^2\omega^2}$ = impedance of the circuit.

NOTE. — From an inspection of Fig. 14 it is evident that two or more reactances, and therefore two or more inductances, in series may be combined by simple addition, as is the case with resistances. The resultant impedance of two or more impedances in series is, however, the vector sum.

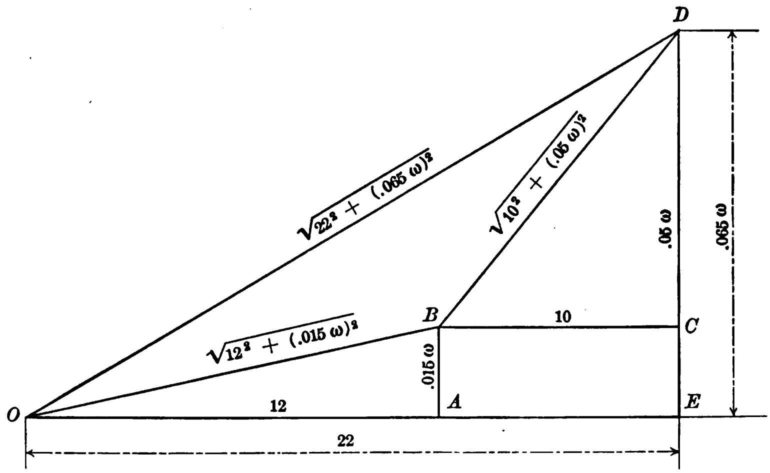



FIG. 14

**17.** Two coils are connected in series; the first has a resistance of 12 ohms and an inductance of .12 henry, the second a resistance of 26 ohms and an inductance of .06 henry. What must be the periodicity of an electro-motive force of 400 volts that produces a current of 5 amperes through these coils?

**18.** An electro-motive force with a periodicity of 100 is impressed upon a circuit with a resistance of 200 ohms. Plot a curve of impedance as the inductance varies from 0 to .4 henry.

**19.** An electro-motive force with a periodicity of 100 is impressed upon a circuit with an inductance of .2 henry. Plot a curve of impedance as the resistance varies from 0 to 200 ohms.

**20.** Plot 2 current curves: one when an electro-motive force of 2000 volts with a periodicity of 80 acts upon a circuit with a constant inductance of .5 henry and a resistance varying from 0 to 200 ohms; the other when the same electro-motive force acts upon a circuit with a constant resistance of 50 ohms and an inductance varying from 0 to .5 henry.

Fig. 15 represents the electro-motive force triangle of a circuit of resistance $R$ and capacity $C$ acted upon by an electro-motive force of periodicity $n = \frac{\omega}{2\pi}$. $OB$ is the effective electro-motive force in phase with the current and numerically equal

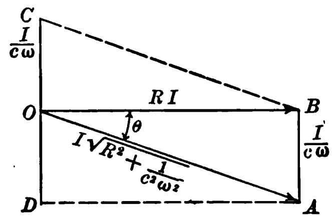



FIG. 15

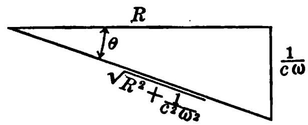



FIG. 16

to $RI$; $OC$ is the reactive electro-motive force 90° ahead of the current in phase and equal to $\frac{I}{C\omega}$, while $OD = BA$ is that component of the impressed electro-motive force required to overcome $OC$. The total impressed electro-motive force is accordingly the resultant $OA$ of $OB$ and $OD$ and equals $I\sqrt{R^2 + \frac{I}{C^2\omega^2}}$.

Evidently the current will lead the impressed electro-motive force in phase by an angle $\theta$ whose tangent is $\frac{I}{C\omega} \div RI$, or $\theta = \tan^{-1}\frac{1}{Cr\omega}$.

In the same way as in the inductive circuit the ohmic triangle is as shown in Fig. 16, excepting that $\theta$ is now the angle of lead. The impedance is $\sqrt{R^2+\left(\frac{1}{C\omega}\right)^2}$.

**21.** What is the impedance in a circuit of 12 ohms resistance and 5 ohms capacity reactance?

**22.** What is the capacity in Problem 21 if the periodicity is 125?

**23.** If an electro-motive force of 100 volts is impressed on the circuit of Problem 21, what current is produced?

**24.** By what angle does the current lead the electro-motive force in Problem 23?

**25.** Draw the sine curves of the various electro-motive forces and current in the case of Problems 21 to 24.

**26.** When the periodicity is 100, what is the impedance of a line containing 200 ohms resistance and 5 microfarads capacity?

**27.** In Problem 26 what capacity will double the impedance?

**28.** With a periodicity of 54, what is the impedance of a circuit of 5000 ohms resistance and 1 microfarad capacity?

**29.** If the resistance in Problem 28 is doubled, how and by what per cent will the impedance be affected?

**30.** If the resistance of Problem 28 is reduced one half, how and by what per cent will the impedance be affected?

**31.** If the capacity in Problem 28 is doubled, how and by what per cent will the impedance be affected?

**32.** If the capacity in Problem 28 is reduced one half, how and by what per cent will the impedance be affected?

**33.** If the frequency in Problem 28 be changed to 34, what value must be given to the resistance, the capacity remaining 1 microfarad, to keep the impedance at 5804 ohms?

**34.** With a periodicity of 100, find graphically the impedance of a circuit of 175 ohms resistance and 5 microfarads capacity.

**35.** Find graphically the resistance of a circuit of 13 microfarads capacity whose impedance is 216 ohms at a frequency of 72.

**36.** In a circuit of 4 microfarads capacity the periodicity is 80. Draw a curve of impedance as the resistance varies from 0 to 1200 ohms.

**37.** In a circuit of 600 ohms resistance the periodicity is 80. Draw a curve of impedance as the capacity varies from 2 to 8 microfarads.

**38.** A constant difference of potential of 2000 volts at a frequency of 125 is maintained between the terminals of a circuit. Draw 2 curves of current: one with a constant resistance of 100 ohms and a capacity varying from 0 to 100 microfarads; the other with a constant capacity of 10 microfarads and a resistance varying from 0 to 1000 ohms.

When both inductive and capacity reactance electro-motive forces are large enough to be taken into account, one, due to

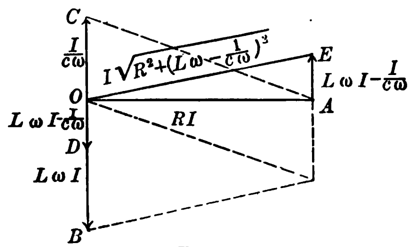



FIG. 17

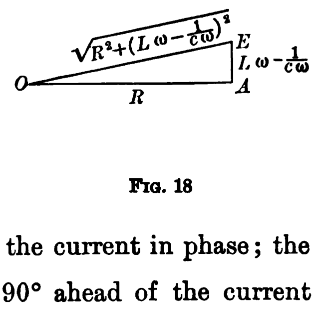



FIG. 18

inductance $= L\omega I$, will be 90° behind the current in phase; the other, due to capacity $= \frac{I}{C\omega}$, will be 90° ahead of the current in phase.

These are shown by $OB$ and $OC$, respectively, in Fig. 17, where $OA$ represents the phase relations of the current and the magnitude of the ohmic drop. The algebraic sum of the two reactance electro-motive forces, $L\omega I$ and $\frac{I}{C\omega}$, is the net or effective reactance electro-motive force represented by $OD$. It is clear that either $L\omega$ or $\frac{1}{C\omega}$ may be the greater, the net reactance accordingly being either 90° behind or 90° ahead of the

current in phase. To overcome the reactance there must be impressed upon the circuit an electro-motive force $AE$, equal and opposite to the net reactive electro-motive force $OD$, while to overcome the resistance there must be an electro-motive force $RI$ in phase with the current; hence the total impressed electro-motive force is represented in magnitude and phase by $OE$, the resultant of $OA$ and $AE$, equal to $I\sqrt{R^2+\left(L\omega-\frac{1}{C\omega}\right)^2}$.

Fig. 18 shows the corresponding ohmic triangle. The tangent of the angle of lag or lead is evidently $\frac{L\omega-\frac{1}{C\omega}}{R}$, the current lagging or leading according as $L\omega$ or $\frac{1}{C\omega}$ is the greater.

The impedance of a circuit containing inductive and capacity reactance is expressed by $\sqrt{R^2+\left(L\omega-\frac{1}{C\omega}\right)^2}$. Plainly, where $L\omega=\frac{1}{C\omega}$, or where $LC\omega^2=1$, the impedance equals the resistance.

**39.** What is the impedance of a circuit of 14 ohms inductive reactance, 5 ohms capacity reactance, and 12 ohms resistance?

**40.** With an impressed electro-motive force of 100 volts, what current will flow in the circuit of Problem 39?

**41.** Does the current of Problem 40 lag or lead, and by what angle?

**42.** Draw the sine curves of the various electro-motive forces and current in Problems 39 to 41.

**43.** With a periodicity of 200, what is the impedance of a circuit of 10 ohms resistance, .3 henry inductance, and 2 microfarads capacity?

**44.** With a periodicity of 80, what is the impedance of a circuit of 180 ohms resistance, 5 henrys inductance, and .76 microfarad capacity?

**45.** What are the phase relations of the current and electro-motive force in the circuit of Problem 44?

**46.** All other conditions in Problem 44 remaining the same, what value of $L$ will make the impedance 180 ohms?

**47.** What current will flow in a circuit of 5 ohms resistance, .01 henry inductance, and 100 microfarads capacity, if 1200 volts with a frequency of 120 are impressed upon its terminals?

**48.** In Problem 47 will the current lag or lead, and by what angle?

**49.** A circuit in which the frequency is 50 contains 500 ohms resistance and 1 henry inductance. Draw a curve of the impedance as the capacity varies from 0 to 20 microfarads.

**50.** A circuit in which the frequency is 50 contains 500 ohms resistance and 10 microfarads capacity. Draw a curve of the impedance as the inductance varies from 0 to 5 henrys.

**51.** A circuit in which the frequency is 50 contains 25 henrys inductance and 10 microfarads capacity. Draw a curve of the impedance as the resistance varies from 0 to 1000 ohms.

**52.** An electro-motive force of 2300 volts is impressed upon a circuit. Draw 4 curves of current: the first with 100 ohms resistance, 1 henry inductance, a frequency of 25, and the capacity varying from 0 to 100 microfarads; the second with 100 ohms resistance, 25 microfarads capacity, a frequency of 25, and inductance varying from 0 to 10 henrys; the third with 1 henry inductance, 25 microfarads capacity, a frequency of 25, and resistance varying from 0 to 1000 ohms; and the fourth with 100 ohms resistance, 1 henry inductance, 25 microfarads capacity, and the frequency varying from 0 to 50.

**53.** An electro-motive force of 100 volts, with a periodicity of 80, is impressed upon a circuit of 10 ohms resistance, 20 microfarads capacity, and .2 henry inductance, all in series. What is the voltage between the terminals of the condenser, and what that between the extremities of the inductance?

**54.** Draw the voltage curves at the terminals of the capacity and of the inductance under the conditions of Problem 53 as the capacity varies from 0 to 100 microfarads.

**55.** Draw the above curves under the conditions of Problem 53 as the inductance varies from 0 to 1 henry.

**56.** Draw the above curves under the conditions of Problem 53 as the resistance varies from 0 to 1000 ohms.

**57.** Draw the above curves under the conditions of Problem 53 as the periodicity varies from 0 to 400 cycles per second.

When a circuit divides into several parallel branches, each possessing resistance and reactance, it is evident that there might be substituted for the multiple impedances

$$\sqrt{R_1^2+\left(L_1\omega-\frac{1}{C_1\omega}\right)^2},\ \sqrt{R_2^2+\left(L_2\omega-\frac{1}{C_2\omega}\right)^2},$$

etc., an equivalent impedance $\sqrt{R^2+\left(L\omega-\frac{1}{C\omega}\right)^2}$, without change of the main current, either in magnitude or phase.

The joint impedance of parallel branches may be quite easily obtained in the following manner:

In Fig. 19 let $AOB$, $AOC$, and $AOD$ represent the electromotive force triangles of 3 parallel branches in which flow

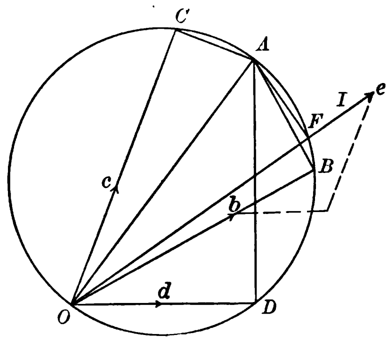



FIG. 19

currents $I_1$, $I_2$, and $I_3$. $R_1$, $R_2$, and $R_3$ are the resistances of the 3 branches. The first has a net inductive reactance $=L_1\omega$, the second a net capacity reactance $=\frac{1}{C_2\omega}$, and the third a net inductive reactance $=L_3\omega$. We then have the impressed

electro-motive force represented by $OA$, the effective electro-motive forces $R_1I_1$, $R_2I_2$, and $R_3I_3$ represented by $OB$, $OC$, and $OD$, and the reactive electro-motive forces $L_1\omega I_1$, $\frac{I_2}{C_2\omega}$, and $L_3\omega I_3$ represented by $AB$, $AC$, and $AD$. Dividing $OB$ by $R_1$, $OC$ by $R_2$, and $OD$ by $R_3$, we get the separate currents represented by $Ob = I_1$, $Oc = I_2$, and $Od = I_3$. The vector sum $Oe$ of $I_1$, $I_2$, and $I_3$ is $I$, the current in the main line before branching, and the current that would flow if a single conductor of equivalent impedance was substituted for the parallel branches. $OF = RI$ is the effective, and $AF = I\left(L\omega - \frac{1}{C\omega}\right)$ the reactive, component of the impressed electro-motive force $E$. $\frac{OF}{I} = R$ is the resistance, and $\frac{AF}{I} = \left(L\omega - \frac{1}{C\omega}\right)$ is the reactance of the equivalent circuit. The phase relations of the various currents may be obtained either analytically by the method of Chapter XIV or with a protractor from the graphical diagram. This method is of course applicable to a circuit of any number of branches, each containing either inductance or capacity, or both.

**58.** An alternating current of 25 amperes, with a periodicity of 150, flows in a circuit closed in one place by parallel branches, one of 4 ohms resistance and .003 henry inductance, the other of 12 ohms resistance and 200 microfarads capacity. Determine the impressed electro-motive force, the equivalent impedance of the parallel branches, the phase relations and magnitude of the component currents, the resistance and reactance of the equivalent circuit, and the phase relation of the main current.

**59.** Two branches, one of 4 microfarads capacity and 500 ohms resistance, and the other of 1 henry inductance and 500 ohms resistance, have impressed upon their terminals an alternating electro-motive force of 1000 volts, with a periodicity of 79.58 cycles per second. What is the equivalent resistance, reactance, and impedance of the circuit, and what the main current in magnitude and phase?

**60.** An alternating current of 2 amperes flows in a circuit of 500 ohms resistance, the frequency being 79.58. Draw curves representing the varying values of the difference of potential at the terminals of the circuit, and varying angle of lead as the capacity varies from 1 to 16 microfarads.

**61.** An electro-motive force of 11,000 volts, with a periodicity of 50 cycles per second, is impressed upon a circuit of 4 parallel branches: one of 200 ohms resistance and 1 henry inductance; a second of 500 ohms resistance and 10 microfarads capacity; a third of 250 ohms resistance, 3 henrys inductance, and 6 microfarads capacity; and the fourth of 600 ohms resistance, 1 henry inductance, and 2 microfarads capacity. What current will flow in each branch, what are the resistance, reactance, and impedance of the equivalent circuit, and what are the magnitude and phase relations of the main current?

**62.** Alternating currents of 1 ampere each, with a periodicity of 39.79, flow in 2 parallel branches, one of 40 microfarads capacity and 10 ohms resistance, and the other of .4 henry inductance and 10 ohms resistance. Determine the main current in phase and magnitude, the impressed electro-motive force, and the resistance, reactance, and impedance of the circuit equivalent to the combination.

# CHAPTER XVII

## DIRECT CURRENT ARMATURES

The fundamental equation of the continuous current dynamo is $E = 2\,nN\Phi 10^{-8}$, where $E$ is the electro-motive force in volts generated in one circuit of the armature; $n$, which may be called the periodicity, is the number of revolutions per second multiplied by the number of pairs of pole pieces; $N$ is the number of conductors in series on the armature, counted from positive to negative brush, and $\Phi$ is the magnetic flux in maxwells or C.G.S. lines of force from each pole piece that are effectively cut by the moving conductors. For practical purposes it may be convenient to express the magnetic flux in megamaxwells or millions of maxwells, in which case the fundamental equation assumes the form $E = .02\,nN\Phi$.

### PROBLEMS

**1.** What electro-motive force is generated by a 4 pole continuous current dynamo having 160 conductors upon a parallel wound drum armature, 5,000,000 maxwells from each pole piece effectively cut by the moving conductors, and a speed of 900 revolutions per minute?

*Solution.* One hundred and sixty conductors upon the face of the armature, with parallel grouping, give, with a 4 pole machine, $N = 40$; 900 revolutions per minute, or 15 revolutions per second, with a 4 pole machine, give $n = 30$; 5,000,000 maxwells are 5 megamaxwells. Hence,

$$E = .02 \times 30 \times 40 \times 5 = 120 \text{ volts.}$$

**2.** In a certain dynamo there are 200 conductors in series in each armature circuit, and the periodicity is 5. The effective

flux density at the gap is 6500 gausses, and the area of the pole faces is 308 square centimeters each. What is the electro-motive force?

**3.** Where $\Phi$ is 3.45 megamaxwells, $n$ is 6, and $N$ is 128, what is $E$?

**4.** In an 8 pole dynamo where the flux density in the gap is 7000 gausses, the area of a pole face is 714.3 square centimeters, the conductors in series per armature circuit are 400, and the electro-motive force is 500 volts. What is the speed of rotation?

**5.** If the dynamo in Problem 4 has an armature resistance of .1 ohm and an output of 50 kilowatts, by what per cent must the flux density be increased to maintain a terminal voltage of 550?

**6.** A 4 pole dynamo has a speed of 500 revolutions per minute, the flux density is 6.8 kilogausses over an area per pole of 824 square centimeters, and each of its armature circuits has 168 conductors in series. What is the electro-motive force?

**7.** A railway generator is required to give a terminal difference of potential of 550 volts, with a full load output of 550 kilowatts. It has 10 poles and a speed of 120 revolutions per minute. Each of its 10 armature circuits has a resistance of .16 ohm. What electro-motive force must be generated at full load?

**8.** What is the magnetic flux per pole in Problem 7, with 110 conductors in series per armature circuit?

**9.** An 8 pole lighting generator built for an output of 200 kilowatts has 76 conductors in series, and runs at 200 revolutions per minute. At no load the voltage is 110. What is the flux density in the gap with a polar area of 780 square centimeters?

**10.** The armature resistance of above generator is .003 ohm. With a full load output at 125 volts, what will be the flux density in the gap?

**11.** There are 8 armature circuits in the generator of Problem 10. With a full load current density of 230 amperes per square centimeter, what is the necessary area of an armature conductor?

**12.** A 6 pole dynamo has an armature wound with 6 circuits, each of .063 ohm resistance. The flux per pole is 7.8875 megamaxwells, and the speed 300 revolutions per minute. It is required to give a terminal electro-motive force of 220 volts, with a full load of 150 kilowatts. What is the number of conductors in series in an armature circuit?

**13.** It is desired to build a generator with an electro-motive force of 250 volts to run as nearly as possible at 500 revolutions per minute; the flux per pole is 2.7 megamaxwells, the armature has a drum winding with 2 circuits, each of 192 conductors in series. What number of poles will require the least variation from desired speed, and by what per cent must the speed be changed from 500 to give the exact voltage?

**14.** A 6 pole generator has a speed of 600 revolutions per minute. What is its periodicity?

**15.** The terminal electro-motive force of the generator of Problem 14 is 600 volts, with an output of 50 kilowatts. The armature is wound as a series drum of 2 circuits, each of 216 conductors in series, and of .2 ohm resistance. The contact resistance of each of the sets of brushes is .006 ohm. What electro-motive force must be generated in each armature circuit?

**16.** The area of a pole face of the generator in Problem 15 is 760 square centimeters. What is the flux density in the air gap?

**17.** What size wire, B.S.G., should be used in the armature to give a current density as near 315 amperes per square centimeter as possible?

**18.** A 2 pole direct current generator has an induced electro-motive force of 250 volts, with an output of 1.15 kilowatts. There are 448 conductors on the armature, and the speed is 1900 revolutions per minute. What is the magnetic flux through the armature?

**19.** Assuming a single magnetic circuit and a leakage coefficient of 1.35 for the dynamo of Problem 18, what is the flux density in the yoke connecting the poles if it has a rectangular area 12 by 16.5 centimeters?

**20.** A 10 pole generator, running at 80 revolutions per minute, has a multiple drum winding of 10 circuits, each containing 128 conductors in series. The flux through each magnet core is 21.4 megamaxwells at full load. Assuming a leakage coefficient of 1.125, what is the full load generated electromotive force?

**21.** In the generator of Problem 20 the section of an armature conductor is .27 by 1.25 centimeters. What is the current density with an output of 300 kilowatts at 315 volts?

**22.** If the terminal full load electro-motive force of above generator is 315 volts, what is the resistance of the armature?

**23.** What is the mean length of a turn of the armature winding if the above resistance is at 35° C.?

**24.** The no load electro-motive force of the above generator is 300 volts. If at this voltage the leakage coefficient is reduced to 1.12, and the cross section of the magnetic path in the magnet frame is 800 square centimeters, what is the density?

**25.** In above generator what is the cross section of magnetic path in the armature core, the no load flux density being 11,500 gausses?

**26.** A 6 pole dynamo has the magnetization of its armature reversed at the rate of 13.75 cycles per second. What is the speed?

**27.** The generator in the preceding problem delivers a current of 800 amperes with an output of 10 kilowatts. The armature resistance is .0025 ohm from positive to negative brushes. The contact resistance of the brushes is .03 ohm per square inch, and there are in all 30 brushes, each with an area of 2 square inches. What electro-motive force must be induced in the armature?

**28.** In the generator above described there are 18 conductors in series in each armature circuit. What is the amount of the magnetic flux entering the armature from any pole?

**29.** If the same generator have a no load electro-motive force of 15 volts, what is the magnetic flux in a cross section of the yoke, assuming a leakage coefficient of 1.125?

**30.** The generator above is multiple wound with 6 armature circuits. If the current density in the armature is 250 amperes per square centimeter, what is the area of an armature conductor?

**31.** At full load a motor absorbs 150 kilowatts of electrical energy with 500 volts at the brushes. The armature resistance is .045 ohm. The contact resistance of the brushes is .03 ohm per square inch, and there are 16 brushes each 1 inch thick and 1¼ inches wide. There are 4 poles and 160 conductors in series per circuit. What is the magnetic flux per pole at a speed of 450 revolutions per minute?

# CHAPTER XVIII

## ALTERNATING CURRENT ARMATURES

When dealing with the alternating current dynamo the fundamental equation demands several numerical coefficients not required in the case of the direct current dynamo.

The first of these is the ratio of the virtual to the average electro-motive force throughout the half period. With a sine wave of electro-motive force this coefficient becomes 1.11. With peaked waves of electro-motive force it is greater than 1.11, while for flat topped waves it is smaller. The value 1.11 will be assumed in the following problems.

Another, called the breadth coefficient $q$, depends upon the armature winding, and while in some modern ironclad alternators it is unity, in the older surface wound armatures and in modern ironclad armatures with distributed windings its value is always somewhat less than unity. When other values are not specified, unity will be assumed.

In accordance with the above assumptions, the fundamental equation for alternators becomes $E = 2.22\, nNq\Phi 10^{-8}$, where $\Phi$ represents the magnetic flux per pole in maxwells. Expressing $\Phi$ in megamaxwells, we have $E = .0222\, nNq\Phi$.

### PROBLEMS

**1.** A certain alternator with a periodicity of 50 has 2.4 megamaxwells per pole effectively cut by 280 conductors in series. What is the value of the electro-motive force?

**2.** A 20 pole alternator runs at 240 revolutions per minute with 636 conductors in series on the armature. The

area of a pole face is 645 square centimeters and the gap density 6.56 kilogausses. What is the electro-motive force?

**3.** A 12 pole alternator has 176 turns in series on its armature and is driven at 600 revolutions per minute. At each pole piece the air gap has an area of 130 square centimeters and a density of 7500 gausses. What electro-motive force is induced?

**4.** A 3 phase 18 pole alternator with 324 conductors per circuit generates an electro-motive force of 1330 volts at a periodicity of 62. What is the magnetic flux per pole?

**5.** An alternator with 120 poles, running at 80 revolutions per minute, delivers 125 amperes with an output of 1500 kilowatts. With 2880 conductors, all in series, upon the armature, what is the gap density with a polar area of 330 square centimeters?

**6.** Keeping the flux per pole constant in the generator of Problem 5, what electro-motive force will be induced if the number of poles is changed to 90 and the number of armature conductors to 2330?

**7.** If in the generator of Problem 5 the number of conductors is made 2330, all else remaining the same, how fast must the machine run to give an electro-motive force of 12,000 volts?

**8.** A 6 pole 60 cycle alternator with 432 conductors upon the armature, all connected in series, delivers a current of 5 amperes with an output of 175 watts. What is the speed? With a leakage coefficient of 1.15, what is the flux in the field cores?

**9.** What electro-motive force is induced in an 8 pole alternator with 1320 conductors in series, an effective gap density of 6300 gausses over a polar area of 385 square centimeters, and a speed of 750 revolutions per minute?

**10.** By what per cent will the electro-motive force of a dynamo be increased if both the number of conductors and the magnetic flux are increased 50%?

11. By what per cent will the electro-motive force of the generator of Problem 9 be changed by increasing the speed by 90 revolutions per minute and the number of armature conductors by 180?

12. A 10 pole alternating current generator runs at 960 revolutions per minute. Its armature is arranged for 4 circuits of 400 conductors in series, giving 1150 volts, or 2 circuits of 800 conductors in series, giving 2300 volts. What is the useful flux per pole?

13. The generator of Problem 12 is rated at 60 kilowatts. If it supplies this power on a non-reactive load, and the resistance between brushes is 1.2 or 4.8 ohms according to the method of connection, by what per cent must the flux be increased to keep the terminal difference of potential at 1150 or 2300 volts?

14. With what size wire, B.S.G., must the generator of Problems 12 and 13 be wound to give a copper area of at least 785 circular mils per ampere?

15. A 16 pole alternator running at 450 revolutions per minute yields an output of 300 kilowatts at 110 volts. There are 24 convolutions in series per circuit. The resistance of the armature and brush contacts being neglected, what is the useful flux per pole?

16. A star connected 3 phase alternator generates a current of 250 amperes in each branch and has a total output of 1000 kilowatts. If the machine has 36 poles, 384 conductors per armature circuit, and an effective flux per pole of 8.2 megamaxwells, what is the speed?

17. What is the output of a single phase alternator supplying the same current per phase at the same terminal electromotive force as that of Problem 16?

18. A 2000 volt delta connected 3 phase alternator with 80 poles runs at 90 revolutions per minute. The flux density in the air gap is 7000 gausses from a pole face of 674 square centimeters area. How many conductors per circuit are needed?

**19.** If the machine of Problem 18 were wound as a 2 phase generator with 2000 volts between the outside wires of the 3 wire line and the speed, flux, and number of poles kept the same, how many conductors per circuit would be needed?

# CHAPTER XIX

## THE WINDING OF ARMATURES

The winding of ring or Gramme armatures is so simple that the student can easily get a clear conception of the subject from the ordinary text-book.

On the other hand, an understanding of the far more complicated drum winding, especially in the case of direct current armatures, is hardly to be attained without considerable practice in writing winding tables and in drawing winding diagrams, both in end view and in development.

Symmetrical drum windings follow the law of Arnold, which, symbolically expressed, is $y = \frac{1}{p}\left(\frac{C}{2b} \mp a\right)$, where $y$ is the spacing, or the distance expressed in elements of the winding, from one side of an armature coil to the other, an element of the winding being the group of conductors corresponding to one side of an armature coil.

$p$ is the number of pairs of poles.

$C$ is the whole number of conductors on the face of the armature.

$b$ is the number of conductors in an element of the winding.

$a$ is the number of times the current divides at the negative brushes, or is the number of pairs of armature circuits.

Thus in the case of a direct current 2 pole armature with 12 coils of 1 convolution each, we have $p = 1, C = 24, b = 1, a = 1$.

$$y = \frac{1}{1}\left(\frac{24}{2} \mp 1\right) = 11 \text{ or } 13.$$

There are 6 possible windings, 3 with each spacing, the tables for 2 of which are given on the opposite page.

| *Com.* | *B* | *F* | *Com.* | *Com.* | *B* | *F* | *Com.* |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 12 | 12 | 1 | 1 | 14 | 2 |
| 12 | 23 | 10 | 11 | 2 | 3 | 16 | 3 |
| 11 | 21 | 8 | 10 | 3 | 5 | 18 | 4 |
| 10 | 19 | 6 | 9 | 4 | 7 | 20 | 5 |
| 9 | 17 | 4 | 8 | 5 | 9 | 22 | 6 |
| 8 | 15 | 2 | 7 | 6 | 11 | 24 | 7 |
| 7 | 13 | 24 | 6 | 7 | 13 | 2 | 8 |
| 6 | 11 | 22 | 5 | 8 | 15 | 4 | 9 |
| 5 | 9 | 20 | 4 | 9 | 17 | 6 | 10 |
| 4 | 7 | 18 | 3 | 10 | 19 | 8 | 11 |
| 3 | 5 | 16 | 2 | 11 | 21 | 10 | 12 |
| 2 | 3 | 14 | 1 | 12 | 23 | 12 | 1 |

Where the column headed *Com.* gives the commutator bars by number, *B* is the element of the winding passing back from a commutator bar to the other end of the armature and *F* is the element of the winding passing forward to the commutator.

Thus, assuming the conductors to consist of 24 copper bars already in place on the cylindrical surface of the armature and adopting a spacing of 11, commutator bar No. 1 will be connected to conductor No. 1, the back end of conductor No. 1 to the back end of conductor No. 12, the front end of conductor No. 12 to commutator bar No. 12, that to the front end of conductor No. 23, and so on, till the front end of conductor No. 14 is connected to commutator bar No. 1, thus completing the connections. In like manner the connections for a spacing of 13 may be traced out.

If the armature is to be wound with wire by hand, the coils might be wound in the following orders for the two cases:

Winder's Tables

| *B* | *F* | *B* | *F* | *B* | *F* | *B* | *F* |
|---|---|---|---|---|---|---|---|
| 1 | 12 | 13 | 24 | 1 | 14 | 13 | 2 |
| 23 | 10 | 11 | 22 | 3 | 16 | 15 | 4 |
| 21 | 8 | 9 | 20 | 5 | 18 | 17 | 6 |
| 19 | 6 | 7 | 18 | 7 | 20 | 19 | 8 |
| 17 | 4 | 5 | 16 | 9 | 22 | 21 | 10 |
| 15 | 2 | 3 | 14 | 11 | 24 | 23 | 12 |

Thus, assuming that coil 1–12 is wound first, we then turn the armature 180° in its forks and wind coil 13–24, then coils 23–10 and 11–22, and so on. Or the winding may progress in the opposite way around the armature, thus:

| *Com.* | *B* | *F* | *Com.* | *Com.* | *B* | *F* | *Com.* |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 12 | 2 | 7 | 13 | 24 | 8 |
| 2 | 3 | 14 | 3 | 8 | 15 | 2 | 9 |
| 3 | 5 | 16 | 4 | 9 | 17 | 4 | 10 |
| | Etc. | | | | Etc. | | |

And similarly for a spacing of 13. After writing the first two lines of the winding tables the rest can be written directly, the

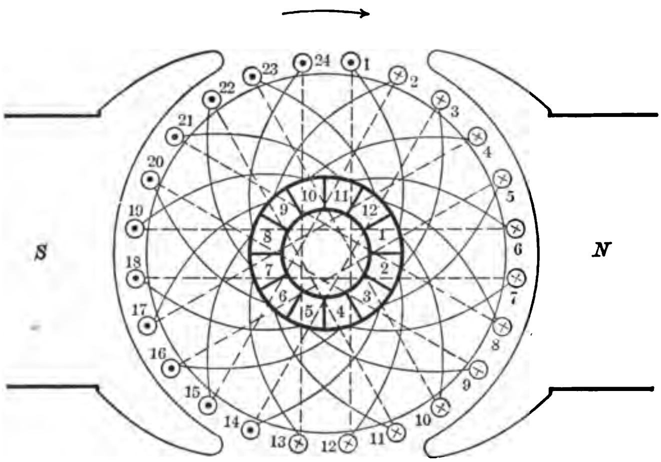



FIG. 20

same amount being added each time in going down the vertical columns. The third possible winding for each spacing would be of the wave type, to be treated later.

The negative value of $a$, resulting in the spacing of 11, is superior to the positive value, resulting in the spacing of 13, in that it requires slightly less wire and fewer crossings at the end of the armature, thus lessening the danger of a failure of insulation in the "heads."

In Fig. 20 is shown the end view and in Fig. 21 the development of the completed winding according to the latter table given for a spacing of 11. Drawing in the pole pieces as shown in the end view and development and assuming a direction of rotation, we may by Fleming's rule determine the directions of the induced electro-motive forces in the conductors that are opposite the pole pieces.

Now placing brushes upon the commutator bars that are connected to coils which lie near the middle of the polar gaps,

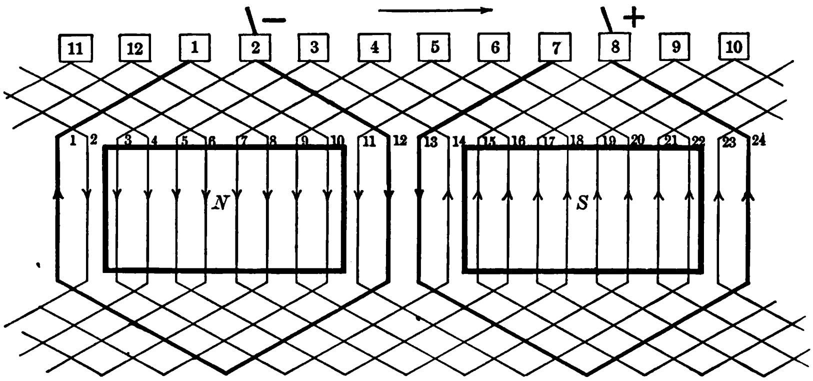



FIG. 21

2 and 8 in Figs. 20 and 21, we can determine the polarity of the brushes, the positive brush being taken as that through which the current leaves the armature.

Beginning at the negative brush, we may trace the current through the various conductors of one of the armature circuits to the positive brush, indicating the direction of the current by arrowheads and by dots and crosses. Returning to the negative brush, we may trace and indicate the direction of the current through the other armature circuit to the positive brush. If the dynamo is multipolar, the above described process may be repeated, starting in turn from each negative brush and passing in both directions to an adjacent positive brush.

## PROBLEMS

**1.** Write the winding table for and draw the end view and development of a 2 pole direct current drum armature with 36 coils, each of 4 convolutions.

SUGGESTION. — There will evidently be 4 conductors in an element and 72 elements in the winding. Instead of attempting to represent the individual conductors in an element as shown in Fig. 22, it is better to represent the elements only as shown in Fig. 23.

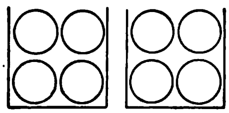


FIG. 22

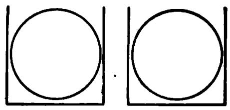


FIG. 23

The formula for this winding is

$$y = \frac{1}{1}\left(\frac{288}{2 \times 4} \mp 1\right) = 35 \text{ or } 37.$$

TABLE

| *Com.* | *B* | *F* | *Com.* |
|---|---|---|---|
| 1 | 1 | 36 | 36 |
| 36 | 71 | 34 | 35 |
| 35 | 69 | 32 | 34 |
| | Etc. | | |

WINDER'S TABLE

| *B* | *F* | *B* | *F* |
|---|---|---|---|
| 1 | 36 | 37 | 72 |
| 3 | 38 | 39 | 2 |
| | Etc. | | |

In drawing the end view it is better for the student to make the connections in the order of the winder's table, imagining as far as possible that he is actually putting wire upon the armature.

Any arrangement of conductors which results in as many armature circuits in multiple as there are pole pieces is termed a *parallel* grouping of the conductors. Any arrangement of conductors which results in but two parallel armature circuits, whatever the number of poles, is called a *series* grouping. It is evident that in two pole machines there is no distinction between series and parallel grouping.

**2.** Make tables and drawings for a 4 pole drum armature with parallel grouping of circuits, there being 48 conductors and 24 commutator bars.

SUGGESTION. — In this case there are 2 pairs of poles, and since the armature current bifurcates twice, *i.e.*, at the 2 negative brushes, $a$ is 2. The formula may then be written $y = \frac{1}{2}\left(\frac{48}{2} \mp 2\right) = 11$ or 13.

**3.** Make drawing for placing the winding in Problem 2 upon an armature with 24 slots.

SUGGESTION. — There are half as many slots as elements; therefore, each slot will receive two elements. Assuming that the elements are symmetrically arranged in slots, 1 and 2 in slot 1, 3 and 4 in slot 2, 5 and 6 in slot 3, etc., the odd numbered elements being at the bottom of the slots, Arnold's formula gives the correct winding for machine made coils; but in the case of hand winding the formula cannot be directly applied. If there were only 24 elements, 1 in each slot, the spacing would be $y = \frac{1}{2}\left(\frac{24}{2} - 2\right) = 5$, and the winder's table:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 6 | 7 | 12 | 13 | 18 | 19 | 24 |
| 3 | 8 | | | Etc. | | | |

Now with the above tables and a complete diagram of the armature slots, themselves numbered and containing the numbers representing the elements of the winding, it is easy to write the winder's table.

*First Layer*

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 11 | 13 | 23 | 25 | 35 | 37 | 47 |

Etc.

*Second Layer*

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 12 | 22 | 24 | 34 | 36 | 46 | 48 | 10 |

Etc.

The end view drawing will be much clearer if the inner and outer layers are drawn in different colors; or in place of the end view the circular development may be used (Fig. 24).

**4.** Write the winding table for and draw the end view and development of the winding of a 6 pole drum with parallel grouping of armature conductors, there being 24 bars to the commutator.

**5.** Write the winding table for a 4 pole armature with parallel grouping of conductors and 48 bars to the commutator.

**6.** The winding in Problem 5 is to be placed by hand upon an armature with 24 teeth. Write the winder's table.

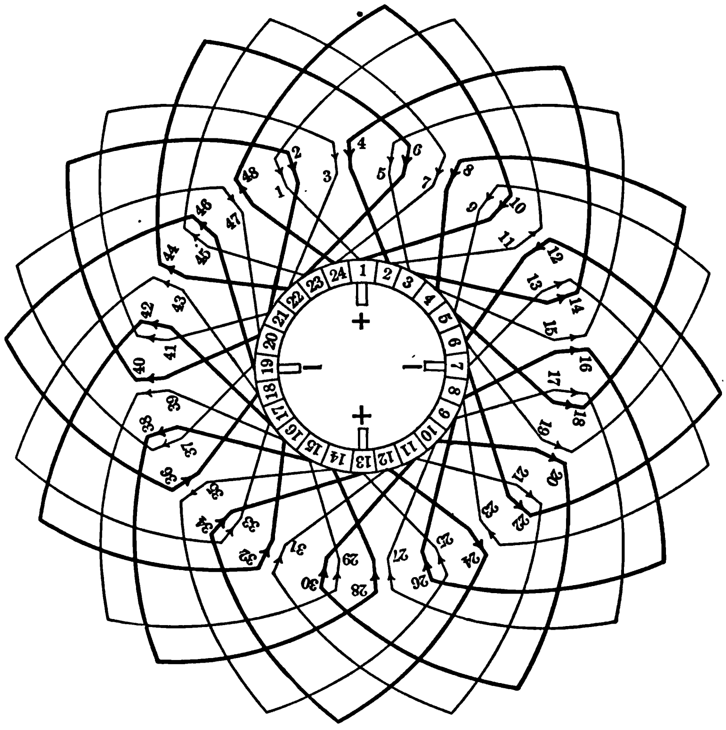


FIG. 24

SUGGESTION. — This will be a case of 4 layer winding with 4 elements in each slot. Following the method described under Problem 3, we have for hand winding:

*First Layer*

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 21 | 25 | 45 | Etc. 49 | 69 | 73 | 93 |

*Second Layer*

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 22 | 42 | 46 | 66 | Etc. 70 | 90 | 94 | 18 |

*Third Layer*

43 63 Etc.

With machine wound coils:

| *First Layer* | | *Second Layer* | |
|---|---|---|---|
| 1 | 22 | 3 | 24 |
| 93 | 18 | 95 | 20 |
| 89 | 14 | 91 | 16 |
| Etc. | | Etc. | |

It will be observed that whatever the position of the brushes upon the commutator, four successive coils always include one from each layer, thus insuring an equality of resistance in the four armature circuits and a constancy of resistance at all times.

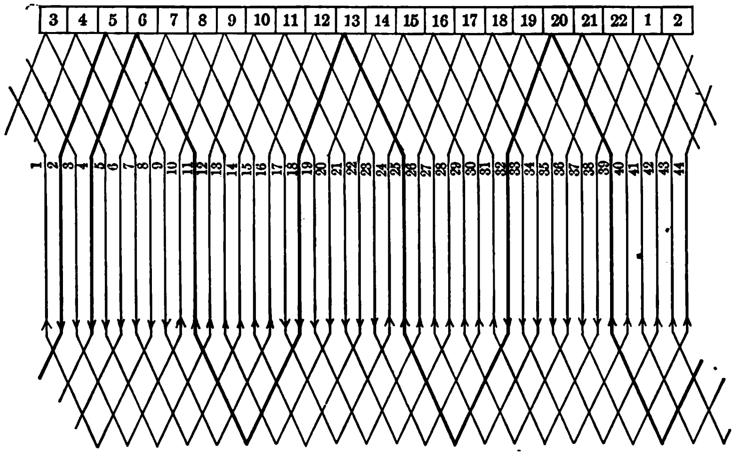



FIG. 25

It is of course evident that in any armature circuit the spacing must be such that two successive elements of the winding will be similarly situated with respect to adjacent, and therefore opposite, field poles. In the windings hitherto considered the connector on the front or commutator end of a coil has lapped back from the *F* element of a given coil to the *B* end of the coil next in order, before or behind, around the surface of the armature. This arrangement is known as *lap winding*, and is clearly shown in the development diagram (Fig. 21).

It is evident, however, that the condition that the electromotive forces in the various elements of an armature circuit

shall be cumulative will be equally well satisfied by connecting the $F$ element of a given coil to the $B$ element of a coil situated about 2 poles in advance of the first coil. This arrangement is known as *wave winding*, and is shown in the development diagram (Fig. 25), which is the development of the winding of Problem 7.

**7.** Write the winding table for and draw the end view and development of a 6 pole wave wound armature having 44 conductors arranged in series grouping.

SUGGESTION. — With series grouping there can be but 2 armature circuits; hence $a = 1$. The formula for spacing becomes $y = \frac{1}{3}\left(\frac{44}{2} \mp 1\right) = 7.$

**8.** Write the table for and draw the development of a 4 pole wave winding having 54 conductors, arranged with series grouping, and connected to a 27 part commutator.

**9.** Write winding tables for a 6 pole wave wound armature with 88 elements connected in 2 parallel circuits.

**10.** In a 6 pole wave wound armature with elements connected in series grouping the spacing is 27. How many elements are there in the winding?

**11.** Write the tables for a 4 pole armature with series grouping, there being 92 conductors arranged in 23 coils, and wound in 2 layers in 23 slots. Draw enough slots in end view to show arrangement of elements in the slots.

**12.** Write tables for and draw end view and development of a 6 pole wave wound armature with series grouping of circuits, there being 124 elements arranged 4 in a slot. Draw teeth and number elements in end view.

Describe the following windings:

**13.** $y = \frac{1}{4}\left(\frac{90}{2} - 1\right).$

*Solution.* An inspection of the formula shows that this is an 8 pole winding with 90 conductors in 45 coils in series grouping.

14. $y = \frac{1}{6}\left(\frac{144}{2} - 6\right).$ 16. $y = \frac{1}{3}\left(\frac{120}{2} - 3\right).$

15. $y = \frac{1}{3}\left(\frac{56}{2} - 1\right).$ 17. $y = \frac{1}{2}\left(\frac{78}{2} - 1\right).$

18. $y = \frac{1}{7}\left(\frac{616}{2 \times 2} - 7\right).$

Cases of mixed lap and wave windings have been designed to meet special conditions. In general they have no particular advantage over simple lap or wave windings.

**19.** Below is given the table for a 4 pole mixed lap and wave winding with series grouping, designed for a particular purpose. There are 84 conductors arranged in 21 slots. Draw the end view and development, place the poles, trace out the currents, and locate the brushes.

$$y = \frac{1}{2}\left(\frac{84}{2}\right) = 21.$$

| *Com.* | *B* | *F* | *Com.* | *B* | *F* | *Com.* | *B* | *F* | *Com.* | *B* | *F* | *Com.* |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 22 | 12 | 43 | 64 | 23 | 45 | 66 | 34 | 3 | 24 | 3 |
| 3 | 5 | 26 | 14 | 47 | 68 | 25 | 49 | 70 | 36 | 7 | 28 | 5 |
| 5 | 9 | 30 | 16 | 51 | 72 | 27 | 53 | 74 | 38 | 11 | 32 | 7 |
| 7 | 13 | 34 | 18 | 55 | 76 | 29 | 57 | 78 | 40 | 15 | 36 | 9 |
| 9 | 17 | 38 | 20 | 59 | 80 | 31 | 61 | 82 | 42 | 19 | 40 | 11 |
| 11 | 21 | 42 | 22 | 63 | 84 | 33 | 65 | 2 | 2 | 23 | 44 | 13 |
| 13 | 25 | 46 | 24 | 67 | 4 | 35 | 69 | 6 | 4 | 27 | 48 | 15 |
| 15 | 29 | 50 | 26 | 71 | 8 | 37 | 73 | 10 | 6 | 31 | 52 | 17 |
| 17 | 33 | 54 | 28 | 75 | 12 | 39 | 77 | 14 | 8 | 35 | 56 | 19 |
| 19 | 37 | 58 | 30 | 79 | 16 | 41 | 81 | 18 | 10 | 39 | 60 | 21 |
| 21 | 41 | 62 | 32 | 83 | 20 | 1 | | | | | | |

The winding of an alternating current armature is very easily represented; care should be taken, however, that the electromotive forces in the various coils of the same armature circuit shall all be cumulative.

Fig. 26 represents the development of a 4 pole single phase winding with 32 conductors in a single circuit.

Fig. 27 represents the development of a 4 pole 2 phase winding with 24 conductors per phase, the 2 circuits being entirely independent.

Fig. 28 represents the development of a 4 pole 3 phase winding with 24 conductors per phase, arranged in star grouping of the phases.

**20.** Draw the development and end view of the winding for a 6 pole single phase armature with 36 conductors in 1 circuit.

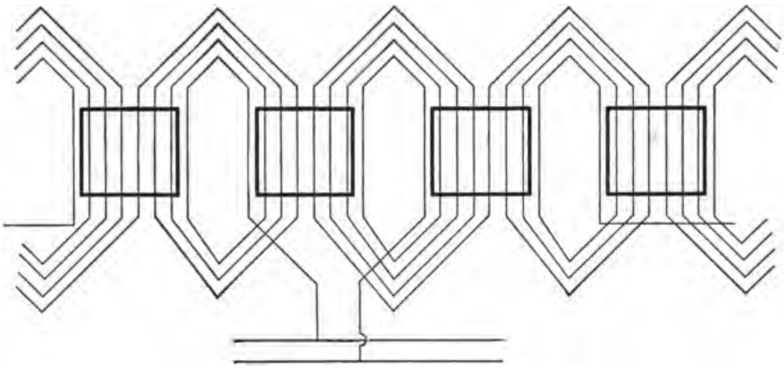


FIG. 26

**21.** Draw the development and end view of the winding for an 8 pole 2 phase armature with 32 conductors per phase.

**22.** Draw the development and end view of the winding of a 6 pole 3 phase armature with 24 conductors per phase, the phases being connected in star grouping.

**23.** Connect the circuits of Problem 23 in mesh grouping.

SUGGESTION. — Number the circuits 1, 2, 3. Let poles be drawn in such position that circuits 1 and 2 at least are generating electro-motive forces; now connect the negative terminals of circuits 1 and 2, the positive terminals of circuits 1 and 3, and the positive terminal of 2 to the negative terminal of 3. These 3 junctions should be connected to the 3 collecting rings.

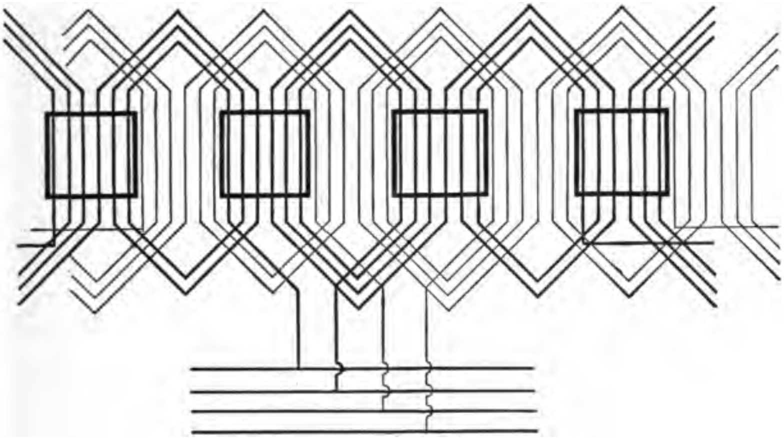


**Fig. 27**

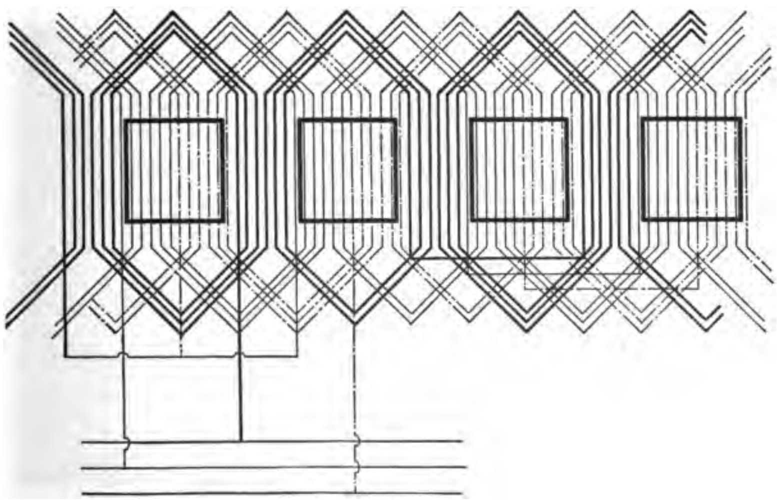


**Fig. 28**

**24.** Draw the development and end view of a 16 pole 3 phase winding with star grouping of the circuits and 576 conductors in all.

SUGGESTION. — As in the case of a direct current armature the various conductors forming one side of a coil may be conceived of as forming an element of the winding, and only these elements need be represented in the drawing.

**25.** Draw the development of a 3 phase 12 pole winding with mesh grouping of the armature circuits and 144 conductors per phase.

**26.** Draw the end view of the winding of a 3 phase 8 pole armature with 48 slots and star grouping of the circuits.

**27.** Draw the development of the winding of a 3 phase 4 pole armature with 8 coils per phase, the circuits being connected in mesh grouping.

# CHAPTER XX

## ARMATURE REACTIONS

If a generator is to maintain its voltage at full load, it must of course generate additional volts enough to force the full load current through the impedance of the armature conductors and brush contacts. If, at the same time, the brushes of a direct current generator are given a forward lead to prevent sparking, the demagnetizing action of the armature currents within the angle of lead weakens the field and increases the leakage coefficient. Since the magnetic flux may be assumed to enter and leave the armature almost entirely by way of its face, it is evident that the magnetizing effect of the conductors depends only on the amount and distribution of the currents in them and not at all on the method of connecting them at the ends of the armature. For our present purpose we may therefore assume the conductors to be connected as shown in Fig. 29, where it is seen that the demagnetizing ampere turns per pole are equal to the product of the number of conductors contained in the angle of lead by the current in each conductor. The currents in the other conductors more or less distort the field but do not materially alter the total flux. Hence the field ampere turns required to force the larger flux needed for the higher electromotive force through the reluctance of the magnetic circuit must be recalculated, and to the result thus obtained must be added the back ampere turns of the armature, the whole being affected by the new value of the leakage coefficient. The above statements apply equally to a direct current motor with a backward lead to the brushes.

## PROBLEMS

**1.** A 10 pole direct current armature has 180 slots with 4 conductors in each. What are the demagnetizing ampere turns

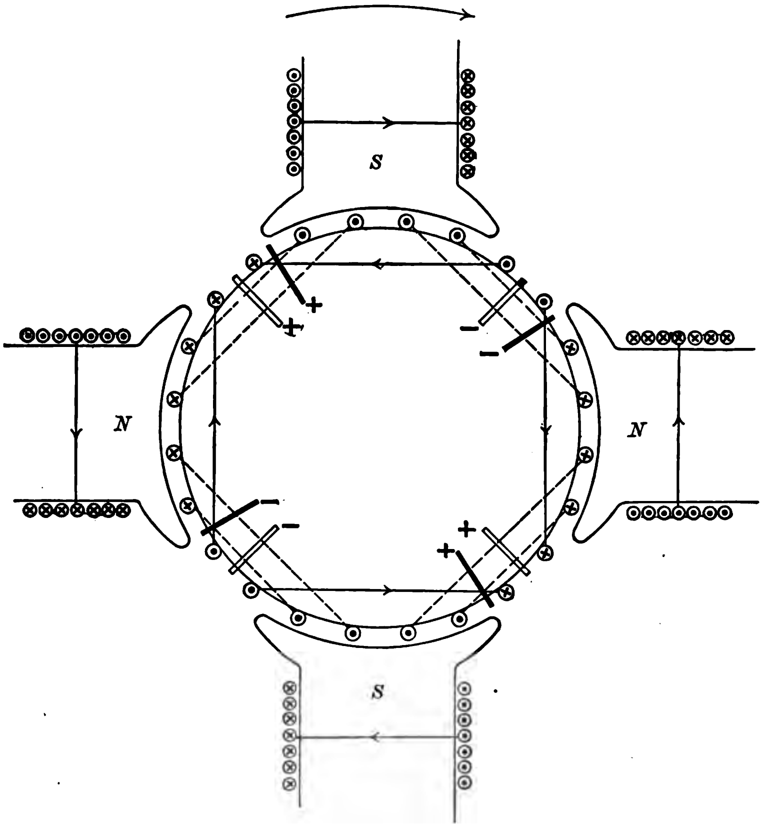



FIG. 29

if each conductor carries 200 amperes and the brushes are advanced $\frac{1}{12}$ of the polar pitch?

**2.** A 2 pole direct current armature has 88 convolutions wound upon it. What are the demagnetizing ampere turns

when the brushes are advanced 10° with a load of 100 amperes?

**3.** A 4 pole generator with 4 armature circuits is wound with 640 conductors in 40 slots. What are the demagnetizing

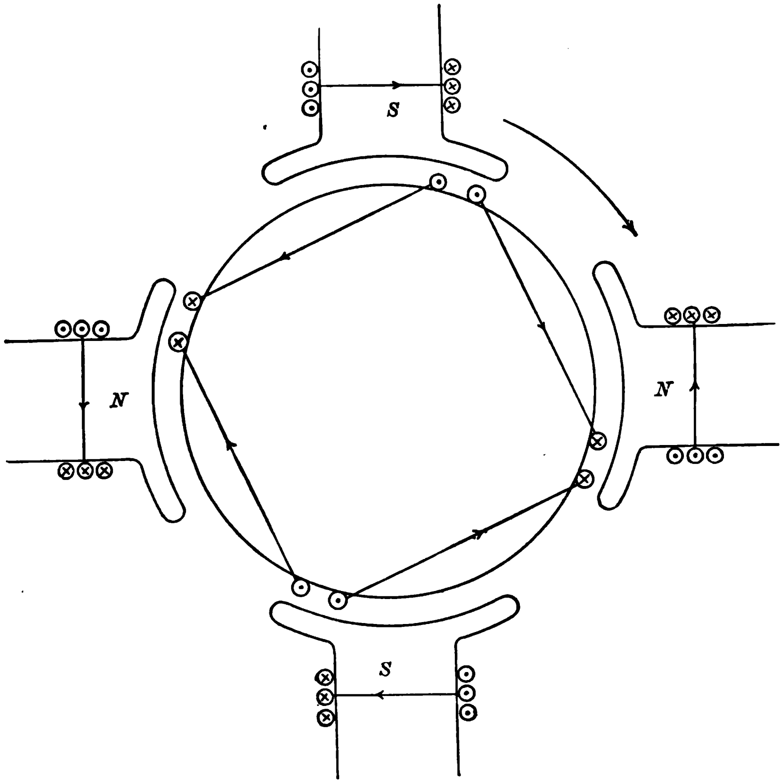



FIG. 30

ampere turns with an output of 120 amperes, the brushes being advanced through an arc of 9°?

**4.** A 12 pole, 1500 kilowatt, 600 volt railway generator is wound with 1392 armature conductors in multiple grouping. What lead of brushes in degrees of arc will produce 2600 demagnetizing ampere turns per pole at full load current?

5. A 12.5 kilowatt input, 4 pole, 250 volt motor with series wound armature has 560 conductors in 35 slots. At full load the brushes are given a backward lead of 7°. How many demagnetizing ampere turns are thereby produced at each pole?

If the current in an alternating current generator lags in phase behind the electro-motive force, then, as shown in Fig. 30, the armature current will set up a magneto-motive force opposing that of the field coils, because each conductor has turned beyond the middle of the polar arc, the position of maximum electro-motive force, before carrying its maximum current, and accordingly the resultant flux is diminished. Evidently a leading current will increase the resultant magneto-motive force. The reactions of a synchronous motor are opposite to those of a generator.

The amount of this reaction for a given angle of lag will depend somewhat upon the distribution of the armature conductors and upon the shape of the field poles, but it may be assumed to vary approximately with the sine of the angle of lag, the net effect being the number of armature ampere turns per pole multiplied by the sine of this angle.

In alternating current armatures also we have not only the resistance drop but a reactive drop due to the inductance of the armature winding. The vector sum of these two, which are of course in quadrature, is the impedance drop and is sometimes quite large.

6. A star connected, 3 phase, 64 pole generator running at 50 cycles per second and supplying 1000 kilowatts at 5000 volts has the armature wound with 32 coils per phase, each of 12 convolutions. At full load with unit power factor, the total drop was observed to be 8% and the measured resistance of the armature was .34 ohm per phase. Determine the value per phase of the drop due to resistance, that due to inductance, and the inductance of the winding.

7. If the armature in Problem 6 is short circuited, by what angle will the current lag behind the electro-motive force?

Consider the self-induction of the armature to be the same as at full load.

**8.** In Problem 7 what will be the demagnetizing ampere turns per pole with full load current flowing in the short circuited armature?

**9.** If the above generator supplies full load current with a lag of 30°, how many demagnetizing ampere turns per pole will it produce?

**10.** A 14 pole single phase generator runs with a power factor of .7 and an apparent output of 180 kilowatts at 30,000 volts. The armature is wound with 14 coils of 156 convolutions each. How many demagnetizing ampere turns are thereby produced?

NOTE. — By apparent output is meant the product of volts and amperes.

# CHAPTER XXI

## FIELD WINDING

The analogue for the magnetic circuit of the law of Ohm is the magnetic flux $\Phi, = \frac{\text{magneto-motive force}}{\text{magnetic reluctance}}$. The magnetic force is expressed by $4\,\pi$ times the absolute current turns, or by $\frac{4\,\pi}{10}$ times the ampere turns. The magnetic reluctance may be expressed by $\frac{\text{reluctivity} \times \text{length of path}}{\text{area}}$, or more usually by $\frac{\text{length of path}}{\text{permeability} \times \text{area}}$; hence $\Phi = \frac{4\,\pi \text{ a. t.}}{10 \frac{l}{\mu A}}$, or a. t. $= \frac{.8\,\Phi l}{A\mu}$, where a. t. signifies ampere turns. The flux density $\frac{\Phi}{A}$ is usually denoted by $B$, so that for any section of a magnetic path we have a. t. $= .8\,B\frac{l}{\mu} = .8\,Hl$, where a. t. are the ampere turns needed to produce the flux of density $B$ gausses through a section of length $l$ and, at that density, permeability $\mu$.

The permeability of air and all materials other than iron and steel used in the construction of dynamos is practically unity; accordingly in them we have a. t. $= .8\,Bl$.

### PROBLEMS

1. How many ampere turns are required to establish a flux of 2.5 megamaxwells across an air gap .5 centimeter long and with an area of 412 square centimeters?

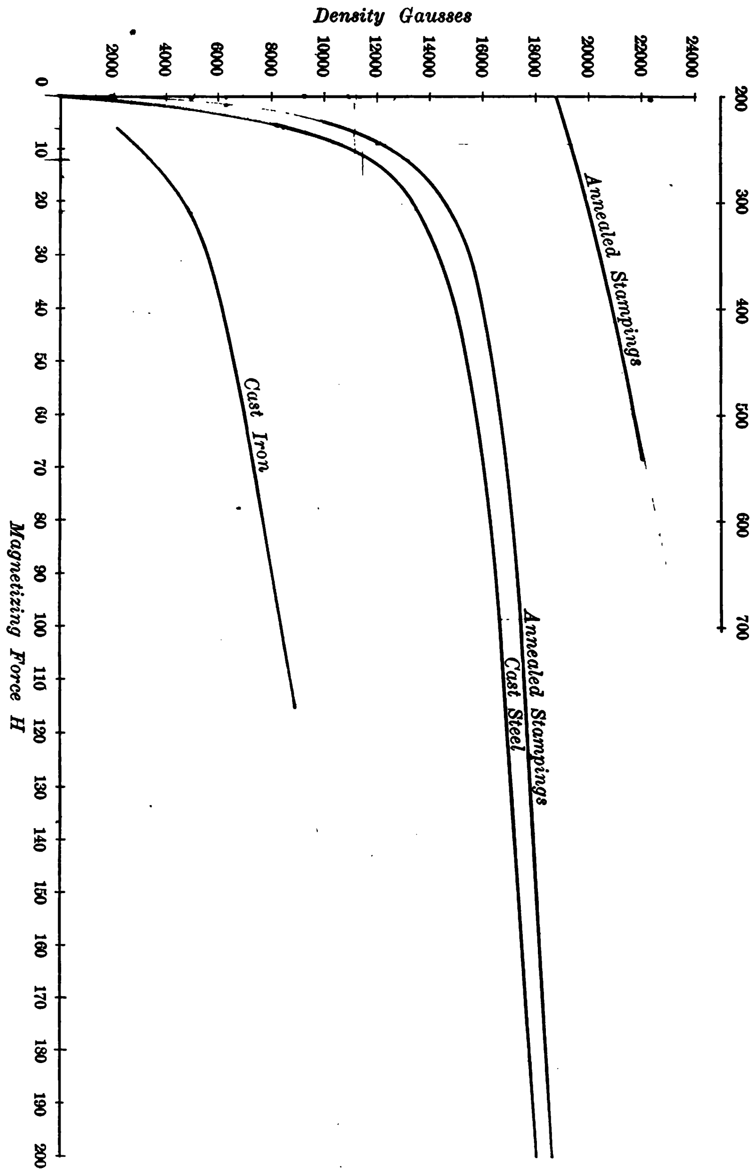

Density Gausses
2000
4000
6000
8000
10000
12000
14000
16000
18000
20000
22000
24000
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
300
400
500
600
700
Annealed Stampings
Cast Iron
Annealed Stampings
Cast Steel
Magnetizing Force H

FIG. 31

**2.** The average length of the flux path through an armature core is 28 centimeters, and its cross section is 100 square centimeters. Selecting a value of $H$ from the curve for stampings (Fig. 31) corresponding to the value of $B$ for the flux of Problem 1, and remembering that only half the given length of path is to be considered as belonging to one pole, and that half the flux from a pole goes in each direction through the armature, calculate the ampere turns per pole required for the armature core.

**3.** There are 9 teeth under each pole, each with an area of 14 square centimeters, and a radial depth of 2.75 centimeters. Determine the ampere turns per pole needed to establish the above flux of 2.5 megamaxwells in the teeth.

**4.** The magnet core of cast steel has a length of 16.5 centimeters and a section of 250 square centimeters. Assuming a leakage coefficient of 1.125, how many ampere turns per pole are needed to set up the required flux in the magnet cores?

**5.** The length of the flux path in the cast-iron yoke ring is 57 centimeters, and its section is 322 square centimeters. Bearing in mind that the leakage coefficient applies to the yoke as well as to the magnet core, and that the flux divides in the yoke as in the armature core; also that each field coil forces the flux through only half the yoke, calculate the ampere turns per pole required for the yoke.

**6.** Summing up the results of the five preceding problems, what number of ampere turns is required for each field coil? If the dynamo has 4 poles, how many ampere turns are required for the whole machine?

Having found the whole number of ampere turns, we may determine the proper size of wire as follows: $R = \frac{\rho L}{A}$, where $R$ is the total resistance of the field, $L$ the total length of the winding, equal to $l$ times the number of turns, $l$ being the length of a turn, $\rho$ the resistance of a centimeter of wire with an area of 1 square millimeter, at the temperature which the

field reaches, and $A$ the area of the wire in square millimeters.

Then $A = \frac{\rho L}{R} = \frac{\rho l \times \text{no. turns}}{R} = \frac{I\rho l \times \text{no. turns}}{E} = \frac{\rho l \times \text{a. t.}}{E}$,

where $E$ is the voltage exciting the field and $I$ the field current. At $0° \text{ C. } \rho = 159 \times 10^{-6}$. The temperature of the coils is likely to reach 60° C., at which $\rho = 197 \times 10^{-6}$ and $A = \frac{197\, l \text{ a.t.}}{10^6 E}$.

Having found $A$, we find from a wire table (page 142) the size of wire to use. If it comes between two standard sizes, in default of having a special wire drawn, the larger of the two must be used. Enough turns must be supplied so that the current density will not exceed the limits of 100 to 160 amperes per square centimeter.

In the United States calculations are frequently made in English dimensions, into which the formula can be changed by a change of the numerical constant.

**7.** Assuming that 14,940 ampere turns are needed for the whole field of above generator, and that we are limited to the standard sizes of wire, what size will be required for the field if the mean length of a turn is 78 centimeters and the electromotive force is 600 volts?

**8.** If the winding space for the field coils is 13.5 centimeters long on each pole piece and the diameter of the insulated wire is .95 millimeter, how deep must the winding be that the current density may not exceed 130 amperes per square centimeter?

**9.** The field coils being joined in series, what will be their resistance at 40° C.? at 60° C.?

**10.** What resistance must be in the field rheostat at 60° C. and at 40° C., in order that in each case there may be 14,940 ampere turns required for the no load electro-motive force of 600 volts to be generated by the armature?

**11.** What is the $I^2R$ loss in the field at 40° C.? at 60° C.?

Field coils require, on the average, for each watt radiated 400 square centimeters divided by the number of degrees

Centigrade difference in temperature between the surface of the coil and the surrounding air.

**12.** If at no load the temperature of the field coils is 40° C. and the machine is generating full load volts, what will be the rise in temperature of the field coils above the surrounding air?

**13.** In the case of the above generator the total armature and brush resistance is 2 ohms, the leakage coefficient at full load is 1.14, and the armature current is 15 amperes. Calculate the ampere turns needed to drive the full load flux through the magnetic circuit.

**14.** There are 2 armature circuits of 710 turns each, and the lead of the brushes with a load of 15 amperes is 10°. Calculate the demagnetizing ampere turns per pole.

**15.** What will be the total ampere turns per pole required at full load?

**16.** Recalculate the size of wire needed in the field.

**17.** Assuming the resistance of the field coils to be that at 55° C., calculate the full load temperature rise above surrounding air with the current density allowed at no load.

**18.** If the generator above treated is compound wound, the shunt coils supplying only the no load ampere turns, what will be the area and number of turns of the series winding for constant potential working, not considering the resistance of the series coils? Allow a density in the series coils of 180 amperes per square centimeter, and assume that $\frac{2}{3}$ of the current flows through the series coils.

**19.** A 3 phase, mesh wound, 760 kilowatt alternator with 64 poles runs at 79.7 revolutions per minute. The terminal voltage is 2200. The pole pieces are 25 centimeters long, parallel to the shaft, with an arc 15 centimeters long, and the air gap is 9 millimeters long. The magnet cores are of cast steel, 14 centimeters long, with an area of 200 square centimeters, bolted on a cast-iron field ring whose external and internal diameters are 565 and 552 centimeters, respectively, and whose length is 32 centimeters, parallel to the shaft. The

armature is wound with 32 coils per phase, each of 12 convolutions. On account of low density we may neglect the reluctance of the magnetic path in the armature core and teeth. Find how many ampere turns are needed at no load, with a leakage coefficient of 1.1.

**20.** The field winding consists of 50 turns per pole of copper ribbon. What is the no load exciting current?

**21.** At full load with unit power factor the drop is 2.81%. What is the armature resistance per phase, hot?

**22.** At full load current with a non-reactive external circuit the drop is 4%. Find the inductance of the armature.

**23.** Calculate the exciting current needed for full load current with the armature short circuited.

**24.** Calculate the full load exciting current on a non-reactive circuit.

**25.** With a power factor of .85, what exciting current will be required to maintain the terminal voltage of 2200 with full load current and inductive load?

## CHAPTER XXII

### THE TRANSFORMER

Fig. 32 represents the diagram of a constant potential transformer with secondary current in phase with the secondary electro-motive force, the transformation ratio being assumed unity. $O\Phi$ represents the flux, produced by the exciting current $On$, composed of magnetizing component $Om$ and hysteretic component $mn$ in quadrature. The alternating flux $O\Phi$ induces the electro-motive forces $e'_1$ and $e_2$, respectively, in primary and secondary coils. The secondary electro-motive force $e_2$ sets up

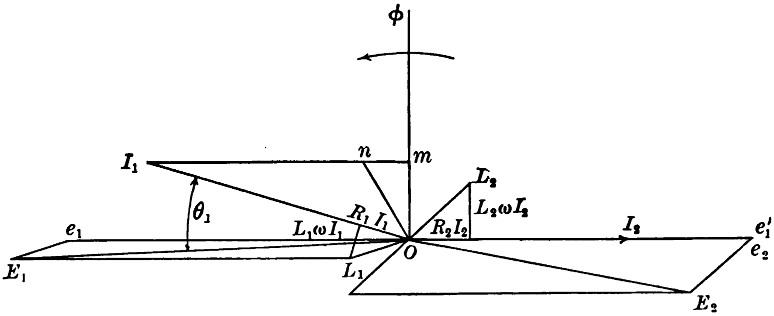



FIG. 32

the current $I_2$, requiring $OL_2$ volts to overcome the impedance of the secondary winding; there results the difference of potential $E_2$ at secondary terminals, the vector difference between $Oe_2$ and $OL_2$. To neutralize the magnetizing action of $OI_2$ amperes in the secondary, there is required $nI_1$ amperes in the primary, which combined with the exciting current $On$ produces the total primary current $OI_1$. To overcome the impedance of the primary, $OL_1$ volts are required. To overcome $Oe'_1$ volts induced in

the primary, $Oe_1$ volts are required; hence there must be $OE_1$ volts impressed upon the primary.

The fundamental equation of the transformer is

$$E = \sqrt{2}\,\pi n N \Phi \times 10^{-8},$$

where $E$ is the electro-motive force induced in $N$ convolutions about an iron core whose total flux $\Phi$ is alternating with a periodicity $n$.

Since in transformers with closed iron magnetic circuits the current and electro-motive force are practically in phase except when the load is very light, we may assume that $E_1$ for the primary coil is equal to the impressed electro-motive force minus the primary ohmic drop, and that $E_2$ for the secondary coil equals the difference of potential at the secondary terminals plus the secondary ohmic drop.

$\Phi$ may of course be expressed as $B\,max \times A$, $A$ being the area of the magnetic circuit and $B\,max$ the maximum value of the induction density during the cycle. The fundamental equation then becomes

$$E = \sqrt{2}\,\pi n N A B\,max \times 10^{-8}, \text{ or } AN = \frac{E \times 10^8}{4.44\, n\, B\,max}.$$

Under practical conditions $E$ and $n$ are fixed and $B\,max$ should be such that with the periodicity employed the loss by hysteresis and eddy currents will not much exceed .015 watt per cubic centimeter.

The approximate flux densities commonly employed with different periodicities are given in the following table:

| *Frequencies* | *B max* |
|---|---|
| 125 | 3000 gausses |
| 100 | 3500 " |
| 80 | 4000 " |
| 60 | 5000 to 6000 " |
| 50 | 6000 to 7000 " |
| 25 | 10,000 to 12,000 " |

An inspection of the fundamental equation shows that, neglecting ohmic drop and magnetic leakage, the ratio of the primary to the secondary voltage, or the ratio of transformation, is the ratio of the primary to the secondary convolutions.

### PROBLEMS

1. A transformer for supplying lamps at 110 volts is connected to primaries at 2200 volts. What is the ratio of transformation?

*Solution.* Neglecting ohmic drop and magnetic leakage, we have

$$\frac{E_1}{E_2} = \frac{2200}{110} = 20.$$

2. If the above transformer has 1200 convolutions in the primary, how many are in the secondary?

3. A step up transformer has 200 convolutions in the primary and a transformation ratio of .1. Assuming 1100 volts to be impressed upon the primary, what is the secondary electromotive force and what the number of secondary convolutions?

4. A system consists of a generator, a step up transformer having 300 primary and 4500 secondary convolutions, a transmission line, and a step down transformer having 5625 primary and 75 secondary convolutions. The step down transformer is to supply current at 390 volts. Neglecting all losses, what must be the electro-motive force of the generator?

5. In the electrical system of Problem 4 assume that there is at full load a 2% loss of pressure in each transformer and a 10% drop on the line. By what per cent must the generator electromotive force be increased to keep the terminal electro-motive force of the secondary at 390 volts?

6. A step down transformer with a transformation ratio of 10 supplies current at 200 volts to a single phase motor of 3 ohms equivalent resistance and 4 ohms equivalent reactance. The primary resistance of the transformer is 2 ohms and the

primary reactance 5 ohms; the secondary resistance is .02 ohm and the secondary reactance .05 ohm. The primary exciting current is .2 ampere and the core loss 250 watts. By means of the transformer diagram determine the magnitude and phase relations of the primary electro-motive force and current.

7. Determine the magnitude and phase relations of the primary electro-motive force and current when the above transformer is furnishing current for an overexcited synchronous motor of 4 ohms equivalent resistance and 3 ohms equivalent reactance.

8. Determine the magnitude and phase relations of the primary electro-motive force and current when the above transformer is furnishing current for 200 16 candle $3\frac{1}{2}$ watt incandescent lamps.

9. A 60 cycle 15 kilowatt transformer for a 2080 volt circuit has 1360 convolutions in the primary and a transformation ratio of 20. The mean length of the magnetic circuit is 179 centimeters and its section 102 square centimeters. Assuming a permeability of 2600, a hysteretic constant of .0015 erg per gauss per cubic centimeter per cycle, and that the eddy current loss is a quarter of the hysteretic loss, what is the exciting current and what is the core loss?

10. What is the no load power factor of the transformer of Problem 9?

11. The primary winding is of No. 9 wire, B.S.G., the mean length of a convolution being 59 centimeters. The secondary has 50 centimeters per turn and is built up of flat strips having an aggregate section of 1.32 square centimeters. At a temperature of 30°C., what is the primary and what the secondary resistance?

12. What is the full load current density in each winding of the transformer in Problem 11?

13. What is the *IR* drop in each of the above coils at full load?

14. What is the total full load copper loss in the above transformer?

**15.** Calculate the efficiency of this transformer at each quarter of full load.

**16.** A 125 cycle, 5 kilowatt, 1100 volt transformer supplies current to a 110 volt lighting circuit. The primary consists of 1060 convolutions of No. 12 wire, B.S.G. The current density in the secondary at full load is .8 that in the primary. The mean length of the magnetic circuit is 98.5 centimeters and its effective area 69 square centimeters. The mean length of a primary convolution is 42 centimeters and of a secondary convolution 36 centimeters. Determine the efficiency at each quarter of full load, at a temperature of 35° C.

**17.** Assuming a constant secondary terminal voltage of 110, determine the primary voltage at each quarter of full load.

**18.** A 25 cycle, 2200 kilowatt transformer to transform from 22,000 to 2200 volts has a double magnetic circuit built up of plates 112 by 57 centimeters, with a rectangular hole 80 by 25 centimeters and a triangular piece 7 centimeters on each of the shorter sides cut from each corner. On each leg of the copper coils enough of these plates are assembled to make an aggregate thickness of iron of 100 centimeters. There are 600 convolutions in the primary coil. What is the maximum value of the flux density?

**19.** Assuming a hysteretic coefficient of .00152 and an eddy current loss half as large as the hysteresis loss, what is the total iron loss at no load?

**20.** Assuming the length of a convolution in both primary and secondary to be 360 centimeters, what will be the full load copper loss with a current density of 225 amperes per square centimeter, the temperature being 45° C.?

NOTE. — The transformer is artificially cooled, and hence this high current density is allowable.

**21.** What will be the efficiency of the transformer of Problem 20 at each quarter of full load?

**22.** To a 400 volt Thompson compensator are connected 40 100 volt incandescent lamps, each requiring 1 ampere.

There are 20 lamps in the first branch, 15 in the second, and 5 in the third. Neglecting all losses, determine and draw diagram showing the current in each coil of the compensator and its direction as compared with the impressed electro-motive force.

**23.** Assuming that the core loss in above compensator is 80 watts and the total resistance of the winding is 1 ohm, what will be the efficiency of the compensator with a load of 60 lamps, evenly distributed on the 4 branches?

**24.** What will be the efficiency of the compensator with the load of Problem 22?

**25.** What is the efficiency of the compensator with 60 lamps evenly divided between 2 branches?

**26.** The transformer described in Problem 16 is used as a booster on an 1100 volt lighting circuit. A lineman thoughtlessly removes the primary fuse at a time when there is a load of 40 kilowatts upon the circuit. Describe the effects and roughly estimate the value of the resulting voltage in the primary of the transformer.

## CHAPTER XXIII

### THE ROTARY CONVERTER

The rotary converter consists of a direct current dynamo with two or more collecting rings connected to symmetrical points in the armature winding. Ordinarily such machines are driven from the alternating current side, and generate direct current. When driven by the direct current they are termed *inverted rotaries.*

In a single phase rotary it is evident that the constant value of the continuous electro-motive force will equal the maximum value of the alternating electro-motive force. Clearly, also, the average power in the primary will equal the average power in the secondary, plus the loss in the rotary. Consider the case of an inverted rotary supplying single phase current to a circuit of unit power factor; assuming the secondary electro-motive force and current to vary harmonically and neglecting losses, we have virtual value of $E_2 = \frac{E_1}{\sqrt{2}}$, since the maximum value of $E_2$ is equal to $E_1$. Hence virtual value of $I_2 = \sqrt{2}\, I_1$, and maximum value of $I_2 = 2\, I_1$. Therefore the value of the secondary power must fluctuate between zero and twice the primary power, with double the frequency of the alternating current.

Evidently, then, the angular velocity of the armature cannot be constant. When the alternating current is zero, all the primary power is spent in accelerating the armature; when the alternating current has its maximum value, half the secondary power is derived directly from the primary, the other half being obtained from the energy previously stored up in the rotating armature.

The field and hysteresis losses in a converter will be those of an alternating dynamo, but the armature $I^2R$ loss will be greater in a single phase and less in a polyphase rotary than those of the ordinary dynamo.

## PROBLEMS

**1.** Neglecting all losses, what is the virtual value of the armature current in an inverted single phase rotary converter, with a direct current input of 50 amperes?

**2.** What is the voltage in the primary of a 3 phase rotary supplying direct current to a street railway feeder at 550 volts?

**3.** If the direct current output in the machine of Problem 2 is 100 amperes, what will be the primary line current in phase with the electro-motive forces?

**4.** A single phase $2\frac{1}{2}$ kilowatt inverted rotary is supplied with current from 110 volt lighting mains. With 1 ampere in the field, the secondary power factor is unity, and 2 amperes are required to drive the armature at full speed with open secondary. The armature resistance between primary brushes is .15 ohm, and the $I^2R$ loss as a converter is 14% greater then as a direct current generator giving the same output at the impressed voltage. What is the efficiency at full load?

**5.** If the 14% increase of $I^2R$ loss in the above armature at unit power factor becomes 37% with a power factor of .8, what will then be the efficiency at full load?

**6.** What will be the secondary current in the above case?

**7.** Neglecting losses, what is the virtual armature current in a 3 phase 10 kilowatt converter delivering direct current at 500 volts?

**8.** What is the speed in revolutions per minute of an 8 pole converter running on a circuit whose periodicity is 60?

**9.** A 16 pole multiple drum converter is driven from a 3 phase, 25 cycle, 320 volt line. Each pole has an area of 752 square centimeters, and the flux density in the gap is 8.27 kilogausses. Determine the number of armature conductors.

**10.** The current density in the armature conductors of the above machine, if run as a 500 kilowatt direct current generator, would be 225 amperes per square centimeter, and the armature $I^2R$ loss would be 38% greater than when equally loaded as a converter. The total length of an armature conductor is 76 centimeters. What is the $I^2R$ loss in the armature with a load of 600 kilowatts if the machine runs at 45° C.?

**11.** Determine the efficiency of the above converter at loads of 200, 400, and 600 kilowatts, assuming a core loss of 7000 watts, 8 horse power lost in mechanical friction, and a loss of 5000 watts in the field.

# CHAPTER XXIV

## THE INDUCTION MOTOR

The fundamental equation of that form of the transformer known as the induction motor is $E = \sqrt{2}\,\pi n q N \Phi \times 10^{-8}$, where $E$ is the virtual value of the electro-motive force induced in one phase of the stator or rotor winding, of $N$ convolutions in series, by the field of $\Phi$ maxwells per pole, rotating at such velocity as to produce the periodicity $n$; $q$ is the breadth coefficient of the coils.

Fig. 33 shows the flux $\Phi$ produced by the exciting current $On$, the resultant of the magnetizing component $Om$ and the hysteretic component $mn$. This flux sweeping round with the

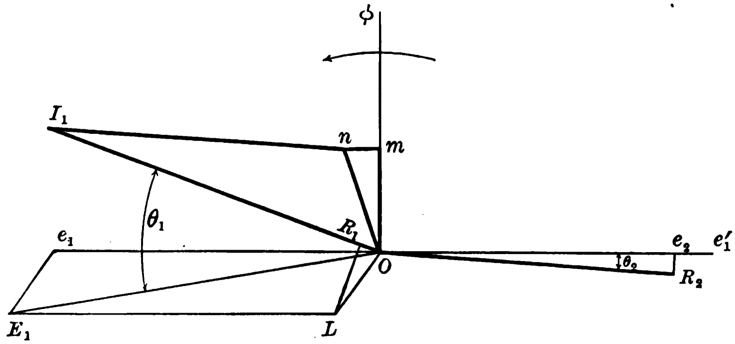



FIG. 33

angular velocity $2\,\pi n$ induces the electro-motive force $e'_1$ in one phase of the stator conductors, and the electro-motive force $e_2$ in one phase of the rotor conductors.

$OR_2$ volts of the electro-motive force $e_2$ are consumed in setting up the current $I_2$ through the rotor resistance $R_2$, and

$e_2R_2$ volts in overcoming the rotor reactance due to rotor slip and leakage.

The current $I_2$ in the secondary demands the component $nI_1$ in the primary, due allowance being made for the ratio of transformation, and this compounded with the exciting current $On$ gives the primary current $OI_1$. To establish this current demands $OR_1$ volts to overcome stator resistance, and $R_1L$ volts to overcome the reactance due to stator leakage. The electromotive force $OL$ required to overcome impedance compounded with $Oe_1$, that required to overcome the counter electro-motive force $Oe'_1$, gives the impressed electro-motive force $OE_1$. The reactance due to magnetic leakage in stator and rotor may be experimentally determined, or it may be approximately calculated by estimating the reluctance of the leakage path between adjacent teeth, and hence the maximum value of the leakage flux encircling a slot when a current whose virtual value is unity flows through $N'$ conductors in that slot. This should be increased by 20% more or less on account of the flux encircling the end connections. The total leakage flux divided by $10^8$ is the inductance in henrys per conductor; this multiplied by the number of conductors in one phase gives the inductance per phase.

From a knowledge of the total power imparted to the rotor, the rotor current may be calculated from the equation watts $= n'N_2I_2\Phi \times 13.28 \times 10^{-8}$, where $N_2$ is the number of convolutions per phase of the rotor winding and $n' = (1-s)n$, $s$ being the per cent of slip. The above equation is derived from fundamental principles on the assumption of the sine distribution of currents and flux about the perimeter of the rotor.

In calculating the magnetizing component of the exciting current, it should be borne in mind that on the assumption of sine distribution of $I$ and $\Phi$ the ratio of the resultant field to the field produced by one phase is half the number of phases.

In the following problems the construction of the clock diagram should proceed along with the numerical calculations.

## PROBLEMS

**1.** A 6 pole 5 horse power induction motor is supplied with 3 phase current at 110 volts and a periodicity of 60. The stator phases are connected in mesh, each phase of the winding being equivalent to 18 coils of 10 convolutions of No. 9 wire, B.S.G., each. The stator has 54 slots .95 centimeter wide and 3.8 centimeters deep; the rotor has 72 slots .8 centimeter wide and 3.2 centimeters deep. The rotor phases are star connected and joined to 3 collecting rings, each phase consisting of 48 bars of copper 1.27 centimeters deep and .475 centimeter wide. The length of laminations parallel to the shaft, excluding ventilating ducts, is 11.5 centimeters; the depth of iron under slots is 2.3 centimeters in both stator and rotor. The external diameter of the rotor is 35.5 centimeters, and the air gap is .125 centimeter. What will be the synchronous speed of the motor?

**2.** Assuming 10% drop in the stator and a breadth coefficient of .95, what is the value of the flux per pole?

**3.** Assuming a sine distribution of the flux over the polar area and, owing to the spread of the lines from the sides of the teeth, assuming the effective polar area to be .6 of the polar pitch multiplied by the length of laminations, calculate the magnetizing component of the exciting current in each phase of the stator winding.

**4.** Calculate the hysteretic component of the exciting current, assuming a hysteretic coefficient of .003.

**5.** Calculate the resistance per phase of the rotor at 40° C., the virtual length of each bar being 40.6 centimeters.

**6.** Assuming the loss by mechanical friction to be .25 horse power, assuming 5% slip, and neglecting the $I^2_2R_2$ loss, calculate the rotor current at full load.

**7.** Recalculate the rotor current, adding the probable $I^2_2R_2$ loss, as deduced from the approximate value of the current above calculated, to the rotor loss.

8. At what per cent slip will the above rotor current be generated?

9. Calculate the self-inductance of a phase of the rotor due to magnetic leakage, assuming the leakage in the slot to extend down 1.2 centimeters from the surface of the rotor.

10. Calculate the rotor reactance.

11. Calculate the lag angle of the rotor current at full load.

12. Determine the ratio of transformation and thence the rotor component of the stator current.

13. Determine the total full load stator current.

14. Assuming the mean length of a convolution of the stator winding to be 82 centimeters, calculate the stator resistance at 50° C.

15. Determine the $I^2{}_1R_1$ loss in the stator.

16. Calculate the reactance of the stator winding, assuming the leakage in the slot to extend inward 1.6 centimeters from the end of the tooth.

17. Complete the clock diagram, determining the exact value of the impressed electro-motive force.

18. Determine the full load power factor from the diagram, and also from the ratio of the real and apparent powers.

19. Determine the full load efficiency.

20. Calculate the no load stator current and power factor, first roughly estimating the primary drop and slip.

21. Draw the complete clock diagram for the above motor at ¼ full load and determine the power factor.

22. Draw the complete clock diagram for the above motor at ½ full load and determine the power factor.

23. Draw the complete clock diagram for the above motor at ¾ full load and determine the power factor.

24. What are the line currents at each of the above loads?

25. What resistance must be inserted in each phase of the rotor circuit in order that it shall exert full load torque when starting with full load current?

**26.** If the above stator had been made with 48 slots, and the supply had been 2 phase at 156 volts, the breadth coefficient being .91, and all conditions not necessarily altered by these changes remaining the same, what would have been the stator current at each quarter of full load?

**27.** In the case of the 3 phase motor treated, draw curves with output in horse power as abscissas and with stator currents, slips, efficiencies, torques, and power factors, respectively, as ordinates.

**28.** Draw curves similar to those of Problem 27 for the 2 phase motor treated.

# CHAPTER XXV

## TESTING OF DYNAMOS

One of the most satisfactory methods of determining the efficiency of dynamos consists in the observation of the no load armature losses due to brush and bearing friction, hysteresis, eddy currents, and air friction. Of these the first two vary directly as the speed, the third as the square of the speed, and the fourth as the cube of the speed. With a given strength of field, we may accordingly write

$$\text{armature losses} = I^2R + (A + a)n + Bn^2 + Cn^3,$$

where $n$ is the speed, $A$, $a$, $B$, and $C$ being constants for the particular machine and strength of field. Moreover $A$ and $C$ are evidently independent of the field, while $a$ varies as the 1.6 power and $B$ as the square of the field strength.

The relative value of the flux with different field currents will evidently vary inversely with the speeds running as a motor without load at a constant armature electro-motive force, allowance being made, if necessary, for the slight $IR$ drop in the armature conductors themselves.

To determine the constants $A$, $a$, $B$, and $C$, run the machine as a motor without load with a constant current in the field, varying the speed by altering the impressed voltage. From the data thus obtained construct a curve with armature watts, minus $I^2R$, as ordinates and speed in revolutions per minute as abscissas. Draw a tangent to this curve through the origin, and the ordinate of this tangent for any particular speed will give the loss due to brush and bearing friction and hysteresis. In like manner construct one or two other curves and tangents, each with

a constant but widely different field current. The values of $A$ and $a$ can now be found, remembering that $A$ is the same for all curves while $a$ varies as the 1.6 power of the field strength. $B$ and $C$ can be easily determined analytically. The values of the constants thus obtained should be verified by seeing that they give points on the curves within the limits of the allowable errors of observation.

Finally the constants $a$ and $B$ may be changed to $h\Phi^{1.6}$ and $e\Phi^2$, where $h$ and $e$ are respectively the hysteresis and eddy current constants and $\Phi$ is the flux in megamaxwells.

Knowing the armature resistance, the total armature losses for any load may now be calculated with considerable precision.

Railway motors are generally tested by mounting the axle gears of two similar machines upon a common shaft, the second machine being driven as a generator by the first. The efficiency of the combination is of course the ratio of the generator output to the motor input. When the resistance drop in the two machines is not excessive, it is customary to consider the efficiency of the motor as the square root of the efficiency of the combination.

The student can easily acquaint himself with the other methods of testing referred to in the following problems.

## PROBLEMS

**1.** An arc light machine was mounted on a Bracket cradle dynamometer. When driven at 800 revolutions per minute, the output was observed to be 6.8 amperes at 2000 volts, and equilibrium was produced by a weight of 57 pounds at the extremity of a 3 foot arm. What was the efficiency of the machine?

**2.** A toy motor was tested with a brake consisting of a piece of twine passing over a groove in the edge of a 5 centimeter brass disk attached to the end of the shaft. To one end of the string a 50 gram weight was hung, the other end being tied to a spiral spring carrying an index. The speed was determined by passing the point of a quill attached to the prong of a vibrating

tuning fork from the center to the periphery of the smoked outer face of the disk and counting the undulations in one or more turns of the spiral thus produced.

The following observations were made: rate of fork 64 periods per second with 16 complete periods in 5 revolutions; extension of spiral spring = force of 78 grams; input of motor 5 amperes at 4 volts. What was the efficiency of the motor?

**3.** A small motor was tested with a brake consisting of a cord and a 6 inch grooved pulley. One end of the cord was attached to a 5 pound weight resting in the pan of a grocer's balance; to the other end was fastened a 1 pound weight hanging free. When running at 1800 revolutions per minute, the apparent weight in the pan was observed to be 2 pounds, the input of the motor being 4 amperes at 100 volts. What was the efficiency of the motor?

**4.** A shunt wound 2 pole dynamo has an armature of .06 ohm resistance. The machine was run as a motor without load, and from the following observations were plotted 3 curves, each with a different strength of field, with revolutions per minute as abscissas and watts delivered to armature, minus $I^2R$ in armature, as ordinates.

WATTS IN ARMATURE MINUS $I^2R$ IN ARMATURE

| *Speed, R.P.M.* | *Field Current* (2.5 Amperes) | *Field Current* (5.0 Amperes) | *Field Current* (7.5 Amperes) |
|---|---|---|---|
| 100 | 22 | 35 | 46 |
| 200 | 46 | 73 | 96 |
| 300 | 71 | 115 | 152 |
| 400 | 98 | 160 | 213 |
| 500 | 127 | 210 | 281 |
| 600 | 159 | 264 | 355 |
| 700 | 194 | 324 | 436 |
| 800 | 232 | 388 | 523 |
| 900 | 274 | 459 | 618 |
| 1000 | 320 | 534 | 722 |
| 1100 | 370 | 616 | 833 |
| 1200 | 425 | 705 | 953 |

The relative values of the total flux with different currents in the field were obtained from the following observations:

| *Volts at Brushes* | *Amperes in Field* | *Revolutions per Minute* |
|---|---|---|
| 60 | 2.5 | 1220 |
| 60 | 5.0 | 660 |
| 60 | 7.5 | 510 |

Determine the constants $A$, $a$, $B$, $C$ for each of the above field currents.

**5.** Calculate the armature losses of the above machine running as a generator at 1150 revolutions per minute with 7.5 amperes in the field and an external load of 50 amperes, there being no lead to the brushes.

**6.** Calculate the efficiency of the above machine when running at 1025 revolutions with 5 amperes in the field, there being 80 volts at the brushes and 70 amperes in the external circuit.

**7.** A small 3 phase alternator was motor driven by the dynamo described in Problem 4 at 1200 revolutions per minute, when the input of the motor armature was observed to be 50 amperes at 150 volts and the field current 7.5 amperes. A wattmeter in one branch of the 3 phase circuit showed 1800 watts, and the alternator fields required 4 amperes at 100 volts. What was the efficiency of the alternator?

**8.** A 2 pole dynamo of the Sprague motor type has each of its fields wound with 1750 convolutions of wire, the 2 coils being connected in series. There are 44 coils, each of 2 convolutions, upon the armature, and with 97 volts at the brushes it was run as a motor without load at 880 revolutions per minute, the field current being 7.2 amperes. With the same voltage and resistance in the field circuit, the following observations were made:

| *Time in Seconds from closing Field Circuit* | *Current Amperes* |
|---|---|
| 1.0 | 4.0 |
| 2.5 | 6.2 |
| 4.5 | 7.0 |
| 5.5 | 7.1 |
| 8 and over | 7.2 |

Determine the time constant of the shunt field circuit and hence the total flux through each leg of the field magnet. From observed voltage and speed and the given armature winding, determine the flux through the armature and hence the leakage coefficient.

**9.** The following observations were taken upon a pair of street railway motors whose axle gears were mounted upon a common shaft, the ratio of reduction being 4.78. Motor No. 1 was run by current from the street railway mains; motor No. 2 was caused to generate, its output being absorbed by a large rheostat. Each motor had a resistance of 1.1 ohms.

| Motor | | Generator | | |
|---|---|---|---|---|
| *Volts* | *Amperes* | *Volts* | *Amperes* | *Armature Speed, R.P.M.* |
| 521 | 20.1 | 393 | 15.2 | 789 |
| 525 | 21.0 | 373 | 14.8 | 768 |
| 518 | 25.4 | 389 | 19.7 | 690 |
| 517 | 27.3 | 392 | 21.6 | 663 |
| 514 | 32.0 | 401 | 25.8 | 612 |
| 518 | 40.2 | 382 | 34.8 | 547 |
| 515 | 50.0 | 360 | 44.0 | 488 |
| 516 | 60.0 | 341 | 52.1 | 445 |
| 516 | 72.8 | 308 | 65.2 | 404 |
| 513 | 75.8 | 302 | 67.8 | 393 |

Reduce the observations to a basis of a constant voltage of 500, the speed being proportional to the voltage, and from the observations construct the following curves:

1. A speed curve having current in amperes for ordinates and a double set of abscissas, one revolutions per minute of the armature and the other speed in miles per hour of a car with 30 inch wheels.

2. A torque curve having current for ordinates and a double set of abscissas, one car axle torque and the other tractive effort.

3. A curve having current as ordinates and efficiency as abscissas.

**10.** A pair of 500 volt railway motors of .75 ohm resistance each were tested in the usual way, the following readings being taken:

| Motor | | Generator | | |
|---|---|---|---|---|
| *Volts* | *Amperes* | *Volts* | *Amperes* | *Armature Speed, R.P.M.* |
| 515 | 9.8 | 50 | 0.0 | 887 |
| 513 | 20.5 | 363 | 12.8 | 525 |
| 520 | 26.3 | 386 | 17.6 | 497 |
| 512 | 31.2 | 388 | 20.3 | 473 |
| 509 | 36.1 | 398 | 25.1 | 454 |
| 509 | 46.8 | 388 | 30.7 | 435 |
| 506 | 56.5 | 377 | 37.9 | 412 |
| 505 | 68.3 | 363 | 45.1 | 398 |
| 490 | 100.5 | 317 | 58.4 | 383 |

Draw the 500 volt speed curve, the reduction ratio being 4.78, and the curve of tractive effort with 30 inch wheels. From these construct curves of efficiency as depending on load and on speed.

# CHAPTER XXVI

## THE TRANSMISSION OF POWER

Power is now transmitted electrically at pressures up to about 60,000 volts, and to distances considerably above 100 miles. Reactances interfere with regulation, and by lowering the power factor diminish the efficiency of the transmission, but both theory and experience show that only the $I^2R$ losses are of material importance.

The student should carefully examine the solution of each of the following problems, and enumerate such general principles as he may be able to deduce therefrom. Unless otherwise specified, unit power factor, a line temperature of 15° C., and in the case of copper a conductivity 97% that of the pure metal, will be assumed.

In the case of polyphase transmission it is well to carry on the computations with reference to a single wire of the line. Thus in three phase lines the voltage per line is $E \div \sqrt{3}$; the resistance, reactance, and impedance per unit length will each be half that of the two wires in a single phase line.

### PROBLEMS

1. Determine the weight of copper per kilowatt delivered by a 10,000 volt generator over a transmission line 10 miles long, at efficiencies varying from 10% to 95%. Construct a curve with efficiencies for abscissas and weights of copper per kilowatt delivered as ordinates.

2. Construct a curve showing the weight of copper per kilowatt delivered at an efficiency of 90% by a 10,000 volt

generator over a transmission line of length varying from 1 mile to 100 miles.

**3.** Construct a curve showing the weight of copper per kilowatt delivered at an efficiency of 90% over a transmission line 10 miles long, with generated voltages varying from 1000 to 10,000.

**4.** If power at the generator costs $20 per kilowatt per annum and copper costs 18 cents per pound, how far will it pay to transmit power from a 10,000 volt generator over a line of 80% efficiency to a point where the selling price of power is $50 per kilowatt per annum? Assume the total cost of the line to be twice that of the copper in it and allow 10% of the cost of the line annually for interest and depreciation.

**5.** What will be the value of power delivered at 30,000 volts over a 75 mile line at 85% efficiency, with power at the generator costing $15 per kilowatt per year and with the same assumptions as to cost of copper, total cost of line, and interest and depreciation as in Problem 4?

**6.** What is the voltage of the generator when power is delivered at 10,000 volts and a periodicity of 30 to a non-reactive load of 200 ohms effective resistance, over a line 10 miles long, of No. 0 B.S.G. aluminum wires hung 30 inches apart? Assume the distributed capacity of the line to be equivalent to that of an equal capacity shunted across the middle of the line.

**7.** Assume the length of the line in Problem 6 to be increased to 25 miles, the terminal voltage to 20,000, and the effective resistance of the non-reactive load to 400 ohms. What will then be the generator voltage?

**8.** Assume the length of line in Problem 6 to be increased to 100 miles, the terminal voltage to 40,000, and the resistance of the non-reactive load to 800 ohms. What will then be the generator voltage?

**9.** Assume the length of line in Problem 6 to be increased to 150 miles, the terminal voltage to 60,000, and the resistance of the non-reactive load to 1200 ohms. What will then be the generator voltage?

**10.** What would have been the generator voltage in Problem 9 if we had assumed the distributed capacity of the line to have been replaced by three condensers, *viz.*, one at each end of the line of $\frac{1}{6}$ the line capacity and one across the middle of the line of $\frac{2}{3}$ the line capacity?

**11.** What is the maximum non-reactive load that can be delivered over the line of Problem 8?

**12.** If 500 kilowatts are delivered at 10,000 volts over the line described in Problem 6, neglecting the line capacity, to an overexcited synchronous motor with a power factor of .8, from generating machinery of 14.22 ohms resistance and .474 henry inductance, what is the generated electro-motive force?

**13.** With the system of Problem 12, draw a curve showing the electro-motive force of the generator at each quarter of full load from no load to 50% overload, the voltage delivered being 10,000 in all cases.

**14.** If in Problem 13 we neglect the capacity of the line and the resistance and inductance of the generating machinery, what is the highest power factor at which 500 kilowatts can be delivered without drop of voltage on the line?

**15.** With a given maximum voltage between lines, what is the relative copper economy of single phase, 2 phase with common return, and 3 phase transmission lines?

**16.** What will be the diameter of copper wires needed to deliver 5000 horse power at a distance of 20 miles with a working pressure of 20,000 volts and a power factor of .9, over a 3 phase transmission line at an efficiency of 85%?

**17.** A 3 phase line 250 miles long consists of 3 No. 000 hard drawn copper wires arranged in the form of a triangle, 15 feet on a side. At the end of this line 10,000 horse power are delivered at a pressure of 70,000 volts and a periodicity of 20, to an inductive load with a power factor of .9. Treating the capacity of the line as in Problem 10 and assuming leakage to be absent, what is the efficiency of the transmission and what the power factor at the generator?

# APPENDIX

## WIRE TABLE, ANSWERS, AND GRAPHICAL SOLUTIONS

## WIRE TABLE

| No. | Diameter | | Area | | Weight | |
|---|---|---|---|---|---|---|
| B.S.G. | mils | mm. | circ. mils | sq. mm. | lbs. per 1000 ft. | kgs. per km. |
| 0000 | 460 | 11.684 | 211,600 | 107.2 | 640.5 | 953.1 |
| 000 | 410 | 10.414 | 168,100 | 85.18 | 508.9 | 757.2 |
| 00 | 365 | 9.271 | 133,225 | 67.52 | 403.3 | 600.1 |
| 0 | 325 | 8.255 | 105,625 | 53.18 | 319.7 | 475.8 |
| 1 | 289 | 7.341 | 83,521 | 42.33 | 252.8 | 376.2 |
| 2 | 258 | 6.553 | 66,564 | 33.73 | 201.5 | 299.8 |
| 3 | 229 | 5.827 | 52,441 | 26.73 | 158.7 | 236.2 |
| 4 | 204 | 5.182 | 41,616 | 21.10 | 126.0 | 187.5 |
| 5 | 182 | 4.623 | 33,124 | 16.79 | 100.3 | 149.2 |
| 6 | 162 | 4.115 | 26,244 | 13.30 | 79.44 | 118.2 |
| 7 | 144 | 3.658 | 20,736 | 10.51 | 62.77 | 93.40 |
| 8 | 128 | 3.251 | 16,384 | 8.303 | 49.48 | 73.80 |
| 9 | 114 | 2.896 | 12,996 | 6.588 | 39.34 | 58.54 |
| 10 | 102 | 2.591 | 10,404 | 5.274 | 31.49 | 46.86 |
| 11 | 91 | 2.311 | 8,281 | 4.195 | 25.10 | 37.35 |
| 12 | 81 | 2.057 | 6,561 | 3.324 | 19.86 | 29.55 |
| 13 | 72 | 1.829 | 5,184 | 2.628 | 15.69 | 23.35 |
| 14 | 64 | 1.626 | 4,096 | 2.077 | 12.40 | 18.45 |
| 15 | 57 | 1.448 | 3,249 | 1.647 | 9.972 | 14.84 |
| 16 | 51 | 1.295 | 2,601 | 1.317 | 7.873 | 11.72 |
| 17 | 45 | 1.143 | 2,025 | 1.026 | 6.130 | 9.121 |
| 18 | 40 | 1.016 | 1,600 | .8109 | 4.843 | 7.207 |
| 19 | 36 | .9144 | 1,296 | .6568 | 3.923 | 5.838 |
| 20 | 32 | .8130 | 1,024 | .5193 | 3.100 | 4.613 |
| 21 | 28.5 | .7239 | 812.3 | .4117 | 2.459 | 3.659 |
| 22 | 25.3 | .6426 | 640.1 | .3244 | 1.938 | 2.883 |
| 23 | 22.6 | .5740 | 510.8 | .2588 | 1.546 | 2.301 |
| 24 | 20.1 | .5105 | 404.0 | .2047 | 1.223 | 1.820 |
| 25 | 17.9 | .4547 | 320.4 | .1624 | .9699 | 1.443 |
| 26 | 15.9 | .4039 | 252.8 | .1282 | .7652 | 1.139 |
| 27 | 14.2 | .3607 | 201.6 | .1022 | .6103 | .9081 |
| 28 | 12.6 | .3200 | 158.8 | .0804 | .4807 | .7153 |
| 29 | 11.3 | .2870 | 127.7 | .0647 | .3866 | .5752 |
| 30 | 10 | .2540 | 100.0 | .0507 | .3027 | .4504 |
| 31 | 8.9 | .2261 | 79.21 | .0402 | .2398 | .3568 |
| 32 | 8 | .2032 | 64.00 | .0324 | .1938 | .2883 |
| 33 | 7.1 | .1803 | 50.41 | .0255 | .1526 | .2271 |
| 34 | 6.3 | .1600 | 39.69 | .0201 | .1201 | .1788 |
| 35 | 5.6 | .1422 | 31.36 | .0159 | .0949 | .1413 |
| 36 | 5 | .1270 | 25.00 | .0127 | .0757 | .1124 |

**WIRE TABLE**

| RESISTANCE | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0° C. | | 20° C. | | 40° C. | | 60° C. | |
| 1000 ft. | km. | 1000 ft. | km. | 1000 ft. | km. | 1000 ft. | km. |
| .0452 | .1482 | .0488 | .1601 | .0524 | .1719 | .0560 | .1838 |
| .0569 | .1866 | .0614 | .2015 | .0660 | .2164 | .0705 | .2314 |
| .0718 | .2354 | .0775 | .2543 | .0832 | .2731 | .0890 | .2919 |
| .0905 | .2969 | .0978 | .3207 | .1050 | .3445 | .1122 | .3682 |
| .1145 | .3755 | .1236 | .4056 | .1328 | .4356 | .1419 | .4657 |
| .1436 | .4712 | .1551 | .5089 | .1666 | .5466 | .1781 | .5843 |
| .1823 | .5981 | .1969 | .6459 | .2115 | .6938 | .2261 | .7416 |
| .2297 | .7537 | .2481 | .8140 | .2665 | .8743 | .2849 | .9346 |
| .2886 | .9469 | .3117 | 1.023 | .3348 | 1.098 | .3579 | 1.174 |
| .3643 | 1.195 | .3934 | 1.291 | .4226 | 1.386 | .4517 | 1.482 |
| .4610 | 1.513 | .4979 | 1.634 | .5348 | 1.755 | .5717 | 1.876 |
| .5835 | 1.915 | .6302 | 2.068 | .6769 | 2.221 | .7235 | 2.374 |
| .7356 | 2.413 | .7945 | 2.606 | .8533 | 2.800 | .9122 | 2.993 |
| .9189 | 3.015 | .9924 | 3.256 | 1.066 | 3.497 | 1.139 | 3.738 |
| 1.154 | 3.788 | 1.247 | 4.090 | 1.339 | 4.394 | 1.432 | 4.697 |
| 1.457 | 4.781 | 1.574 | 5.163 | 1.690 | 5.545 | 1.807 | 5.928 |
| 1.844 | 6.050 | 1.992 | 6.534 | 2.139 | 7.018 | 2.287 | 7.502 |
| 2.334 | 7.657 | 2.520 | 8.270 | 2.707 | 8.883 | 2.894 | 9.495 |
| 2.942 | 9.654 | 3.178 | 10.43 | 3.413 | 11.20 | 3.649 | 11.97 |
| 3.676 | 12.06 | 3.970 | 13.02 | 4.264 | 13.99 | 4.454 | 14.95 |
| 4.721 | 15.49 | 5.099 | 16.73 | 5.476 | 17.97 | 5.854 | 19.21 |
| 5.975 | 19.60 | 6.453 | 21.17 | 6.931 | 22.74 | 7.409 | 24.31 |
| 7.377 | 24.20 | 7.967 | 26.14 | 8.557 | 28.07 | 9.147 | 30.01 |
| 9.336 | 30.63 | 10.08 | 33.08 | 10.86 | 35.53 | 11.58 | 37.98 |
| 11.77 | 38.61 | 12.71 | 41.70 | 13.65 | 44.79 | 14.59 | 47.88 |
| 14.94 | 49.00 | 16.13 | 52.92 | 17.32 | 56.89 | 18.52 | 60.76 |
| 18.72 | 61.40 | 20.21 | 66.32 | 21.71 | 71.23 | 23.20 | 76.14 |
| 23.66 | 77.64 | 25.56 | 83.85 | 27.45 | 90.06 | 29.34 | 96.27 |
| 29.84 | 97.89 | 32.22 | 105.7 | 34.61 | 113.6 | 37.00 | 121.4 |
| 37.82 | 124.1 | 40.84 | 134.0 | 43.87 | 143.9 | 46.89 | 153.8 |
| 47.42 | 155.6 | 51.21 | 168.0 | 55.01 | 180.5 | 58.80 | 192.9 |
| 60.20 | 197.5 | 65.02 | 213.3 | 69.83 | 229.1 | 74.65 | 244.9 |
| 74.86 | 245.6 | 80.85 | 265.3 | 86.84 | 284.9 | 92.83 | 304.6 |
| 96.00 | 313.7 | 103.2 | 338.7 | 110.9 | 363.8 | 118.5 | 388.9 |
| 120.7 | 396.0 | 130.3 | 427.6 | 140.0 | 459.3 | 149.6 | 491.0 |
| 149.4 | 490.1 | 161.3 | 529.3 | 173.3 | 568.5 | 185.2 | 607.7 |
| 189.6 | 622.2 | 204.8 | 672.0 | 220.0 | 721.7 | 235.2 | 771.5 |
| 240.9 | 790.2 | 260.1 | 853.5 | 279.4 | 916.7 | 298.7 | 979.9 |
| 304.8 | 1000 | 329.2 | 1080 | 353.6 | 1160 | 378.0 | 1240 |
| 382.4 | 1255 | 413.0 | 1355 | 443.6 | 1455 | 474.2 | 1556 |

# ANSWERS

## CHAPTER I

1. 30.48 centimeters.
2. 1609.34 meters.
3. 185,319 centimeters.
4. .4115 centimeter.
5. 81,713 square mils, 10,404 circular mils.
6. 211,600 circular mils.
7. 1.016 millimeters.
8. 77.244 cubic centimeters.
9. 506.7 square millimeters.
10. 8154.4 cubic centimeters.
11. 15.43 grains.
12. 437.5 grains.
13. .672 pound.
14. 1.49 kilograms.
15. 62.43 pounds.
16. 31.53 pounds.
17. 6494.9 centimeters.
18. 20,626 feet.
19. 10.24 pounds.
20. 262 pounds.
21. 2.02 pounds.

## CHAPTER II

1. 2 amperes.
2. .25 ampere.
3. 5 ohms.
4. .25 ohm.
5. 16 volts.
6. 30 volts.
7. 11 ohms.
8. 8.45 volts.
9. 40 amperes.
10. 25 amperes.
11. 100 volts.
12. 4 volts.
13. 20 ohms.
14. 57.5 ohms.
15. .75 ampere.
16. 12 volts.
17. .19 ampere.
18. 54.17 ohms.
19. 100 ohms.
20. .000002 ampere.
21. 52 volts.
22. 3 amperes.
23. .00394 ampere.
24. 70 amperes.
25. 22,104 amperes.

## CHAPTER III

1. .00000159.
2. 9.56.
3. 35.5 ohms.
4. 989.34 feet.
5. 10%.
6. .000037 ohm.
7. .0000783 ohm.
8. .326 ampere.
9. 28.52%.
10. .00000965.
11. 57.98.
12. .2665.
13. .006184 ohm.
14. 1.19 ohms.
15. .1146 ohm.
16. 1592.4 centimeters.
17. .195 centimeter.
18. .1377 centimeter.
19. .0337 ampere.
20. .0000029.
21. 17.44.
22. 1.8 ohms.

## CHAPTER IV

1. 4 amperes.
2. 33 ohms.
3. 1024 volts.
4. .00005 ampere.
5. .4 ampere.
6. 6.875 amperes.
7. 10 amperes.
8. 78 volts.
9. 8 amperes.
10. $33\frac{1}{3}\%$.
11. .4 ampere.
12. .2 ampere.
13. .133 ampere.
14. .0667 ampere.
15. .346 ampere.
16. .32 ampere.
17. .292 ampere.
18. .8 ampere.
19. .5 ohm.
20. 42.5 volts.
21. A–B, .058 ohm; A–C, .111 ohm; A–D, .110 ohm; B–C, .054 ohm; B–D, .054 ohm; C–D, .111 ohm; B connected to the common junction.
22. .892 ampere.
23. .359 ampere.
24. .374 ampere.
25. .291 ampere.
26. .191 ampere.
27. See page 157.

## CHAPTER V

1. 5 mhos, .2 ohm.
2. 2.05 mhos, .488 ohm.
3. 284 volts.
4. .8 ampere; 12 volts.
5. 1 ampere; 2 amperes.
6. $B/A$.
7. 1.904 amperes.
8. $33\frac{1}{3}\%$.
9. $33\frac{1}{3}\%$.
10. 8 cells.
11. .5 ampere.
12. .38 ampere.
13. .218 ampere.
14. .177 ampere.
15. .0909 ampere.
16. .0457 ampere.
17. .55 ampere, all in series.
18. 1.1 amperes; 2 groups in parallel, each of 10 in series.
19. 2.2 amperes; 4 groups in parallel, each of 5 in series.
20. 2.75 amperes; 5 groups in parallel, each of 4 in series.
21. 5.5 amperes; 10 groups in parallel, each of 2 in series.
22. 11 amperes; all in parallel.
23. Series 45.45 amperes; multiple 24.39 amperes.
24. 1 ohm.
25. .025 ohm.
26. .00125 ampere.
27. .101 ohm.
28. Main current increased 98%; galvanometer current decreased 98%.
29. 9.9 ohms.
30. 333.33 ohms; 30.303 ohms; 3.003 ohms.
31. Decreased 37.46%.
32. $s_1 = 4.004$ ohms, $s_2 = 40.77$ ohms, $s_3 = 493.33$ ohms, $r_1 = 36.36$ ohms, $r_2 = 403.64$ ohms, $r_3 = 3556$ ohms.

## CHAPTER VI

2. 20.76 volts.
3. 2.56 miles.
4. 176 volts at 7th car.
5. 474.23 volts at 4th car.
6. 24 volts, generator and line; 25 volts, lamps.
7. 2528 volts generated with line at 60°; 2502 volts generated with line at 0°.
8. 9.69 volts drop to 10th group.
9. 11.98 volts drop to 10th group.
10. See page 158.
11. 444.38 volts at 4th car.
12. 617.26 volts.
13. 533.87 volts at station.
14. 468.04 volts at last car.
15. 123 volts at last car.
16. 487.6 volts at opposite corner.
17. 5.4 volts at opposite corner.
18. 79.23 volts at car on opposite corner.
19. 454.47 volts at car on opposite corner.

## CHAPTER VII

1. 10 kilowatts; 13.45 horse power.
2. 4.5 watts.
3. 55 watts; 3.44 watts per candle.
4. 200 watts.
5. 300 watts.
6. 15.41 horse power.
8. 86.96%.
9. 50%.
10. 80%.

11.

| | |
|---|---|
| Series | 82.3 watts, |
| 12 groups | 57.6 " |
| 8 " | 38.4 " |
| 6 " | 26.2 " |
| 4 " | 13.7 " |
| 3 " | 8.2 " |
| 2 " | 3.84 " |
| multiple | .99 watt. |

12.

| | |
|---|---|
| Series | 11.75 watts, |
| 12 groups | 23.04 " |
| 8 " | 23.04 " |
| 6 " | 19.00 " |
| 4 " | 11.70 " |
| 3 " | 7.50 " |
| 2 " | 3.69 " |
| multiple | .978 watt. |

13.

| | |
|---|---|
| Series | 14.30%, |
| 12 groups | 40.00%, |
| 8 " | 60.00%, |
| 6 " | 72.72%, |
| 4 " | 85.70%, |
| 3 " | 91.43%, |
| 2 " | 96.00%, |
| multiple | 98.97%. |

14.

| | |
|---|---|
| Series | 14.30%, |
| 12 groups | 40.00%, |
| 8 " | 60.00%, |
| 6 " | 72.72%, |
| 4 " | 85.70%, |
| 3 " | 91.43%, |
| 2 " | 96.00%, |
| multiple | 98.97%. |

15. See page 158.
16. 25%, 4346 watts; 50%, 5390 watts; 75%, 7134 watts; 100%, 9578 watts; 125%, 12,722 watts.

17. 25%, 85.20%; 50%, 90.30%; 75%, 91.30%; 100%, 91.26%; 125%, 90.80%.
18. 69.25 volts.
19. 84.49%.
20. 125.75 volts.
21. 93.04%.
22. 86%.
23. 140.75 volts.
24. Dynamo 7.8%; line 10%; arcs 82.2%.
25. 90.6%.
26. 90.41 amperes.
27. 24.2 amperes.
28. 43.05 amperes.
29. 24.5 horse power.

## CHAPTER VIII

1. .4 gauss.
2. .48 gauss.
3. 2.18 gausses.
4. .1 gauss.
5. 1.136 gausses.
6. .656 gauss.
7. 0.
8. 1.968 gausses.
9. 1.26 gausses.
10. 10 amperes.
11. 1.5 centimeters.
12. .196 gauss.
13. 48° 17′ .5.
14. .137 gauss.
15. .143 ampere.
16. .0716 ampere; a sine galvanometer.
17. .143 ampere.
18. 75.4 gausses.
19. 122.145 gausses.
20. 18,312 gausses.
21. 107.4 gausses.
22. 82.4 gausses.
23. 9.17 gausses.
24. 28.8 maxwells.
25. 13,755 gausses; 43,213 maxwells.
26. 37.7 centimeters.
27. .02512 centimeter.
28. .2528 centimeter.
29. 8 maxwells.
30. 502.4 gilberts.
31. 125.7 gausses.
32. A, 20,000 gausses;
    B, 11,428 "
    C, 12,245 "
    D, 6,000 "
    E, 8,333 "
33. A, 2540 gilberts;
    B, 136.3 gilberts;
    C, 397 gilberts;
    D, 1910.8 gilberts;
    E, 6250 gilberts.
34. 2.38 amperes.
35. 1.06 amperes.
36. 18, 53,280;
    21, 75, 917;
    22, 58,245.
37. $106.56 \times 10^{-8}$ coulomb;
    $151.8 \times 10^{-8}$ "
    $116.5 \times 10^{-8}$ "
38. $6.747 \times 10^{-8}$ coulomb.
39. $166.6 \times 10^{-8}$ coulomb.
40. 3710.5 divisions.

## CHAPTER IX

1. $L_1 = .014$ henry, $L_2 = .00014$ henry.
2. $L_1 = 7.56$ henrys, $L_2 = .0756$ henry.
3. $L_1 = 3.755$ henrys, $L_2 = .03755$ henry.
4. .5445 henry, .0667 henry.
5. $L_1 = .03447$ henry, $L_2 = 13.79$ henrys.
6. $L_1 = .158$ henry, $L_2 = .000253$ henry.
7. $L_1 = 1.257$ henrys, $L_2 = .0503$ henry.

8. 1.672 henrys.
10. See page 157.
11. .825 ampere; 82.43 amperes; 216.15 amperes; 250 amperes.
12. $T = .04$; 7.584 amperes.
13. See page 157.
14. See page 157.
15. See page 157.
16. 17.2 henrys.
17. .000063 henry.
18. .000474 henry.
19. .579 henry.
20. .0049 henry.
21. .0625 henry.
22. .059 henry.
23. .251 henry.
24. $M = .00798$ henry; $L = .1825$ henry.
25. .0019 henry.
26. .704 henry.
27. .0035 henry.

## CHAPTER X

1. 100,000 C.G.S. units.
2. 250.
3. 20 C.G.S. units.
4. .00000006 coulomb.
5. .0000000504 second.
6. 7958 C.G.S. units or .00884 microfarad.
7. 61 plates.
8. 9804.6 square centimeters.
9. 5.22 microfarads.
10. 316,626 C.G.S. units, .3519 microfarad.
11. 39 plates.
12. .010063 microfarad.
13. 7.95 centimeters.
14. 159; 17.37 centimeters.
15. 1.002 microfarads.
16. 2.445 microfarads.
17. .186 microfarad.
18. See page 157.
19. See page 157.
21. $13.817 \times RC$ seconds.
22. .0004424 coulomb; .001554 coulomb; .001987 coulomb.
23. .3894 ampere; .1116 ampere; .0034 ampere.
24. $T = .00025$; $q = .0000237$ coulomb; $i = .0552$ ampere.
25. See page 157.
26. See page 158.
27. See page 158.
28. See page 159.
29. .00543 microfarad.
30. .00927 microfarad.
31. .521 microfarad.
32. 1.199 microfarads.
33. 9 microfarads.
34. .923 microfarad.
35. 2.22 microfarads.

## CHAPTER XI

2. .000167 volt; current leaves the lead.
3. .00178 volt; current towards the iron.
4. .00156 volt.
5. See page 159.
6. See page 158.
7. See page 159.
8. .00000116 ampere.
9. 835° C.
10. .00000477 ampere.
11. 27.3 centimeters.
12. 977° C., 458° C.

## CHAPTER XII

2. Hydrogen, .0000416 gram per second; chlorine, .001468 gram per second.
3. 1.18 grams. 4. 33.87 amperes; 22.758 grams.
5. 49 minutes, 1 second. 6. 10 hours, 47 minutes, 22 seconds.
7. 17.83 amperes, 2.17 amperes; oxygen .00148 gram per second, hydrogen .000185 gram per second.
8. 538.86 tons. 9. 204 days, 3 hours, 39 minutes, 32 seconds.
10. 6.53. 11. 1.2 amperes.

## CHAPTER XIII

1. See page 160. 5. See page 161. 9. See page 159.
2. 7.07 volts. 6. See page 159. 10. See page 161.
3. See page 160. 7. See page 159. 11. See page 159.
4. 3.536 amperes. 8. See page 159. 12. See page 160.
13. See page 161. 14. Virtual value = 71.25 volts; see page 163.
15. Virtual value = 72.4 amperes; see page 161.

## CHAPTER XIV

1. 132.9 volts. 2. 135.3 volts. 3. 121.7 volts.
4. 30°, 193.2 volts; 60°, 173.2 volts; 90°, 141.4 volts; 120°, 100 volts; 150°, 51.8 volts; 180°, 0.
5. 90°. 6. 36° 52′. 7. 62° 43′, 26° 23′. 8. 103.1 volts.
9. 141.8 volts, 81° 56′. 11. 85.4 volts, 114° 11′. 12. 0.
13. 1732 volts; phase angles 90°, 210°, 330°; see Problem 13, Chapter XIII.
14. 1000, 0°; 1414, 45°; 1000, 90°. 15. 707.5 volts.

## CHAPTER XV

1. 1.414 : 1.000. 2. 50 amperes. 3. 0.
4. 13 amperes; its negative is 22° 37′ away from 12 ampere component in phase. 5. 2.65 amperes.
7. Leading current, 18.42 amperes; lagging current, 7.57 amperes.
8. 3.47 amperes. 11. 158.81 amperes, 74° 50′.
9. 14.38 amperes, 76°, 51′. 12. 173.2 amperes.
10. 104.54 amperes, 247° 30′. 13. .174 ampere.

**CHAPTER XVI**

**2.** .003 henry. **3.** 36° 52′. **4.** See page 162. **5.** 5.9 ohms.
**6.** 11.7 amperes. **7.** 200 volts. **8.** 182.3.
**9.** Problem 7, 22° 31′ ; Problem 8, 66° 25′. **10.** .102 henry.
**11.** 18.39 ohms.
**12.** 266.4 ohms.
**13.** 9.165 ohms.
**14.** 8.62 ohms.
**15.** 44.77 ohms.
**16.** 27.4 ohms.
**17.** 62.25.
**18.** See page 163.
**19.** See page 163.
**20.** See page 162.
**21.** 13 ohms.
**22.** 254.6 microfarads.
**23.** 7.69 amperes.
**24.** 22° 37′.
**25.** See page 164.
**26.** 376 ohms.
**27.** 2.18 microfarads.
**28.** 5804 ohms.
**29.** Increased 79.6%.
**30.** Decreased 33.4%.
**31.** Decreased 10.2%.
**32.** Increased 33.2%.
**33.** 3432 ohms.
**34.** 363.2 ohms.
**35.** 133 ohms.
**36.** See page 163.
**37.** See page 163.
**38.** See page 163.
**39.** 15 ohms.
**40.** 6.67 amperes.
**41.** 36° 52′ lag.
**42.** See page 164.
**43.** 23.89 ohms.
**44.** 209 ohms.
**45.** Current leads 30° 30′.
**46.** 5.21 henrys.
**47.** 157.9 amperes.
**48.** Current leads 48° 54′.
**49.** See page 165.
**50.** See page 165.
**51.** See page 165.
**52.** See page 166. **53.** Condenser 990 volts, inductance 1000 volts.
**54.** See page 165.
**55.** See page 165.
**56.** See page 165.
**57.** See page 165.
**58.** E.M.F. = 106.255 volts; resistance = 4.082 ohms; reactance = 1.182 ohms; impedance = 4.25 ohms; $I_1$ = 21.65 amperes, lags 35° 15′ behind the E.M.F.; $I_2$ = 8.105 amperes, leads E.M.F. by 23° 50′; $I$ = 25 amperes, lags 16° 9′ behind the E.M.F.
**59.** Resistance = 353.55 ohms; reactance = 0; impedance = 353.55 ohms; current = 2.828 amperes in phase with the E.M.F.
**60.** See page 167.
**61.** $I_1$ = 29.54 amperes; $I_2$ = 15.86 amperes; $I_3$ = 22.83 amperes; $I_4$ = 7.8 amperes; resistance = 175.2 ohms; reactance = 102.97 ohms; impedance = 203.2 ohms; $I$ = 54.13 amperes, lags 30° 25′ behind the E.M.F.
**62.** $I$ = .19864 ampere in phase with the E.M.F.; impressed E.M.F. = 100.5 volts; resistance = 505.9 ohms; reactance = 0; impedance = 505.9 ohms.

## CHAPTER XVII

2. 40 volts. 3. 53 volts. 4. 187.5 R.P.M. 5. 11.8%. 6. 314 volts.
7. 566 volts. 8. 25.73 megamaxwells. 9. 6.96 kilogausses.
10. 8.21 kilogausses. 11. .87 square centimeter.
12. 96. 15. 609.33 volts. 18. 1.76 megamaxwells.
13. 6 poles, 3.55%. 16. 6186 gausses. 19. 12 kilogausses.
14. 30. 17. No. 6. 20. 324.65 volts.
21. 282.2 amperes per square centimeter. 22. .01 ohm.
23. 291 centimeters. 28. 3.253 megamaxwells.
24. 12,305 gausses. 29. 1.705 megamaxwells.
25. 764 square centimeters. 30. 533 square centimeters.
26. 275 R.P.M. 31. 10.1 megamaxwells.
27. 16.1 volts.

## CHAPTER XVIII

1. 746 volts. 3. 457.1 volts. 5. 7,109 gausses.
2. 2390 volts. 4. 2.98 megamaxwells. 6. 7281 volts.
7. 98.93 R.P.M.
8. 1200 R.P.M.; .07 megamaxwell.
9. 3554 volts. 10. 125%. 11. 27.3%. 12. 1.619 megamaxwells.
13. 5.44%. 14. No. 10. 15. 1.72 megamaxwells.
16. 80 R.P.M. 17. 577.3 kilowatts. 18. 318. 19. 225.

## CHAPTER XIX

2. ONE WINDING TABLE

| *Com.* | *B* | *F* | *Com.* |
|---|---|---|---|
| 1 | 1 | 12 | 2 |
| 2 | 3 | 14 | 3 |
| 3 | 5 | 16 | 4 |

Etc.

4. ONE WINDING TABLE

| *Com.* | *B* | *F* | *Com.* |
|---|---|---|---|
| 1 | 1 | 8 | 2 |
| 2 | 3 | 10 | 3 |
| 3 | 5 | 12 | 4 |

Etc.

5. ONE WINDING TABLE

| *Com.* | *B* | *F* | *Com.* |
|---|---|---|---|
| 1 | 1 | 24 | 2 |
| 2 | 3 | 26 | 3 |
| 3 | 5 | 28 | 4 |

Etc.

7.

| *Com.* | *B* | *F* | *Com.* |
|---|---|---|---|
| 1 | 1 | 8 | 8 |
| 8 | 15 | 22 | 15 |
| 15 | 29 | 36 | 22 |

Etc.

8. ONE WINDING TABLE

| Com. | B | F | Com. |
|---|---|---|---|
| 1 | 1 | 14 | 14 |
| 14 | 27 | 40 | 27 |
| 27 | 53 | 12 | 13 |

Etc.

9.

| Com. | B | F | Com. |
|---|---|---|---|
| 1 | 1 | 16 | 16 |
| 16 | 31 | 46 | 31 |
| 31 | 61 | 76 | 2 |

Etc.

10. 160 or 164.

11. HAND WINDING

| | | | | | |
|---|---|---|---|---|---|
| *First Layer* | 1 | 13 | 25 | 37 | Etc. |
| *Second Layer* | 14 | 26 | 38 | 4 | Etc. |

12. MACHINE WOUND COILS

| | | | | | |
|---|---|---|---|---|---|
| 1 | 22 | 43 | 64 | 85 | 106 |
| 3 | 24 | 45 | 66 | 87 | 108 |

Etc.

HAND WINDING

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| *First Layer* | 1 | 21 | 41 | 61 | 81 | 101 | Etc. |
| *Second Layer* | 105 | 2 | 22 | 42 | 62 | 82 | Etc. |
| *Third Layer* | 86 | 106 | 3 | 23 | 43 | 63 | Etc. |
| *Fourth Layer* | 67 | 87 | 107 | 4 | 24 | 44 | Etc. |

14. Twelve poles, 144 conductors in 72 coils, multiple grouping.
15. Six poles, 56 conductors in 28 coils, series grouping.
16. Six poles, 120 conductors in 60 coils, multiple grouping.
17. Four poles, 78 conductors in 39 coils, series grouping.
18. Fourteen poles, 616 conductors in 154 coils, multiple grouping.

## CHAPTER XX

1. 1200 per pole.
2. 489 per pole.
3. 480 per pole.
4. 3°.23.
5. 272 ampere turns.
6. Drop due to resistance is 39.27 volts; drop due to inductance is 247.9 volts; inductance, .00683 henry.
7. 81°.
8. 2053.
9. 1040.
10. 408.

## CHAPTER XXI

1. 2427 ampere turns.
2. 110 ampere turns.
3. 643 ampere turns.
4. 136 ampere turns.
5. 419 ampere turns.
6. 3735; 14,940.
7. No. 21.
8. 4.66 centimeters.

9. 977.5 ohms at 40° C.; 1045 ohms at 60° C.
10. 80 ohms at 60° C.; 147.5 ohms at 40° C.
11. 278 watts at 40° C.; 297 watts at 60° C.
12. 19°.23 C.
13. 15,992 ampere turns.
14. 591 ampere turns.
15. 4589 ampere turns.
16. No. 20.
17. 29°.5 C.
18. 87 turns per pole of conductor with an area of .0833 square centimeter.
19. 6764 ampere turns.
20. 135.28 amperes.
21. .52 ohm.
22. .0021 henry per phase.
23. 35.4 amperes.
24. 154.42 amperes.
25. 178.92 amperes.

## CHAPTER XXII

2. 60.
3. 11,000 volts; 2000 convolutions.
4. 1950 volts.
5. 15.7%.
6. Current = 4.2 amperes; E.M.F. = 2042.6 volts, current lags 53° 13′.
7. Current = 4.011 amperes; E.M.F. = 1990.3 volts, current leads 33° 25′.
8. Current = 5.727 amperes; E.M.F. = 2023.5 volts, current lags 3° 27′.
9. .16 ampere, 206 watts.
10. .62.
11. Primary, 2.19 ohms; secondary, .00464 ohm.
12. Primary, 106.9 amperes per square centimeter; secondary, 109.24 amperes per square centimeter.
13. Primary, 16 volts; secondary, .67 volt.
14. 214 watts.
15. ¼ load, 94.46%; ½ load, 96.65%; ¾ load, 97.18%; full load, 97.28%.
16. ¼ load, 94.3%; ½ load, 96.4%; ¾ load, 96.8%; full load, 96.8%.
17. ¼ load, 1105.7 volts; ½ load, 1110.6 volts; ¾ load, 1115.4 volts; full load, 1120.3 volts.
18. 10,316 gausses.
19. 12.604 kilowatts.
20. 18.06 kilowatts.
21. ¼ load, 97.56%; ½ load, 98.47%; ¾ load, 98.64%; full load, 98.63%.
22. See page 163.
23. 98.68%.
24. 96.56%.
25. 95.16%.
26. About 3500 volts. The secondary, taking full load current, becomes the primary. This current sets up a very large magnetic flux through the primary, which produces a large electro-motive force.

## CHAPTER XXIII

1. 35.35 amperes in each circuit.
2. 336.8 volts.
3. 94.3 amperes.
4. 84.7%.
5. 83.8%.
6. 40.06 amperes.
7. 10.9 amperes.
8. 900 revolutions.
9. 2688 armature conductors.
10. 5317 watts.
11. 91.5% at 200 kilowatts; 95.2% at 400 kilowatts; 96.3% at 600 kilowatts.

## CHAPTER XXIV

1. 1200 R.P.M.
2. 217,280 maxwells.
3. 4.2 amperes.
4. .45 ampere.
5. .006 ohm.
6. 99.2 amperes.
7. 104.3 amperes.
8. 4.75%.
9. .000036 henry.
10. 000655 ohm.
11. 6° 11′.
12. 7.5; 13.9 amperes.
13. 15.38 amperes.
14. .427 ohm.
15. 303 watts.
16. .832 ohm.
17. 110.26 volts.
18. Cos $\theta = .88$; real power ÷ apparent power = .88+.
19. 81.7%.
20. 4.62 amperes; power factor = .25.
21. .67.
22. .83.
23. .89.
24. 0 load = 8.0 amperes;
 ¼ load = 10.4 amperes;
 ½ load = 14.9 amperes;
 ¾ load = 20.0 amperes;
 full load = 26.6 amperes.
25. .1177 ohm.
26. 0 load, 5.8 amperes;
 ¼ load, 7.2 amperes;
 ½ load, 9.3 amperes;
 ¾ load, 12.8 amperes;
 full load, 16.7 amperes.
27. See page 168.
28. See page 169.

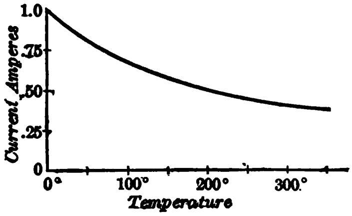
## CHAPTER XXV

1. 70%.
2. 4.31%.
3. 32%.
4. $A = .15$; $C = 6 \times 10^{-8}$;
 field current 2.5 amperes, $a = .07$, $B = .00004$;
 field current 5.0 amperes, $a = .18725$, $B = .0001336$;
 field current 7.5 amperes, $a = .2825$, $B = .000229$.
5. 1041 watts.
6. 82.4%.
7. 80%.
8. $T = 1.24$, flux each leg ≐ 3,436,700 maxwells; leakage coefficient = 1.83.
9. See page 170.
10. See page 170.

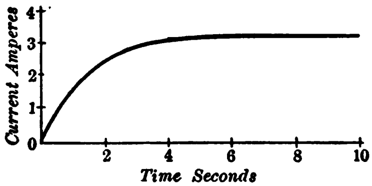
## CHAPTER XXVI

1. See page 167.
2. See page 165.
3. See page 167.
4. 56.1 miles.
5. $22.14.
6. 10,895 volts.
7. 22,230 volts.
8. 48,744 volts.
9. 72,790 volts.
10. 72,837 volts.
11. 2098 kilowatts.
12. 10,000 volts.
13. See page 167.
14. .9.
15. Ratio of weights of copper, single phase 1.000, 2 phase 1.457, 3 phase .750.
16. .268 inch.
17. Efficiency, 87.8%; power factor, .97.

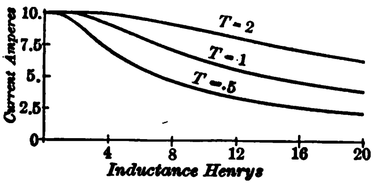
# GRAPHICAL SOLUTIONS



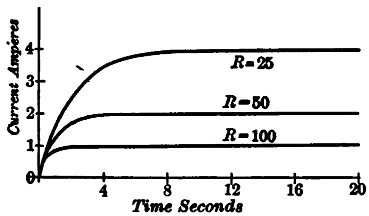

Problem 27, Chapter IV



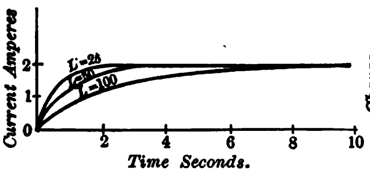

Problem 10, Chapter IX



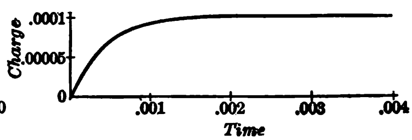

Problem 13, Chapter IX



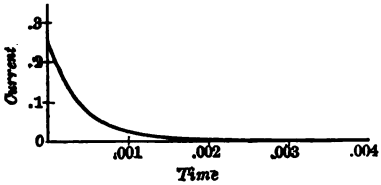

Problem 15, Chapter IX



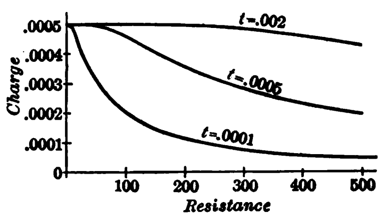

Problem 14, Chapter IX



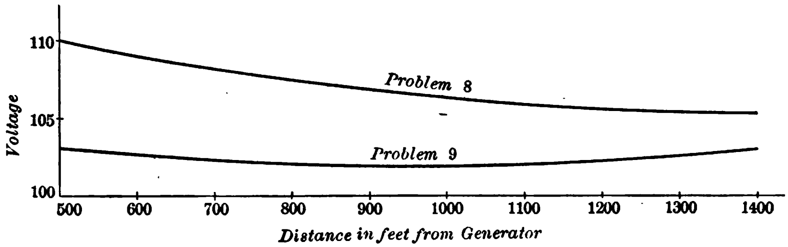

Problem 18, Chapter X



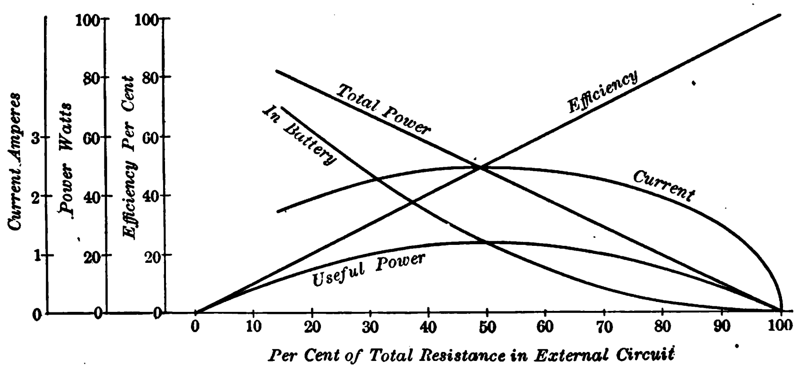

Problem 19, Chapter X



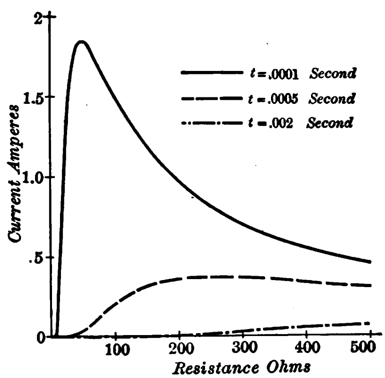

Problem 25, Chapter X



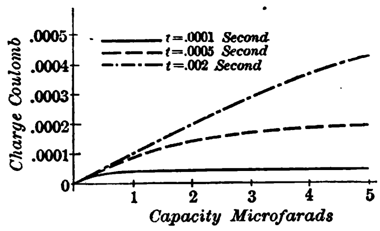

Problem 10, Chapter VI



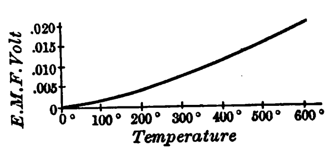

Problem 15, Chapter VII



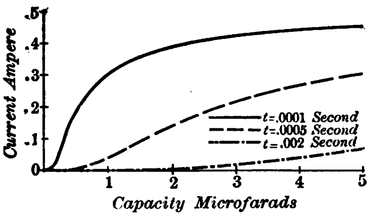

Problem 26, Chapter X



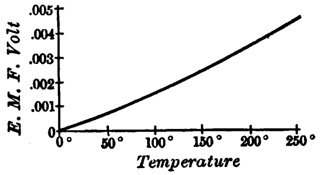

Problem 27, Chapter X



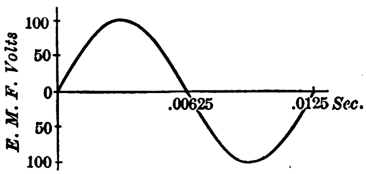

Problem 6, Chapter XI



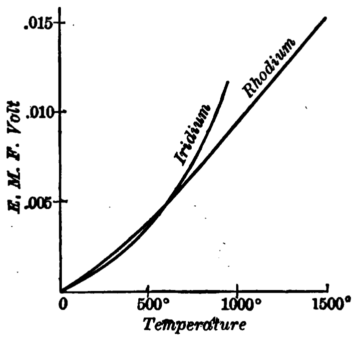

Problem 28, Chapter X



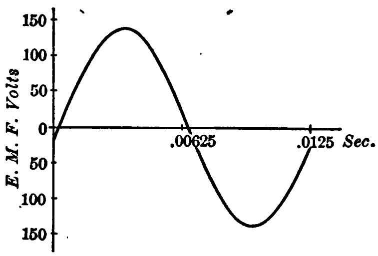

Problem 5, Chapter XI



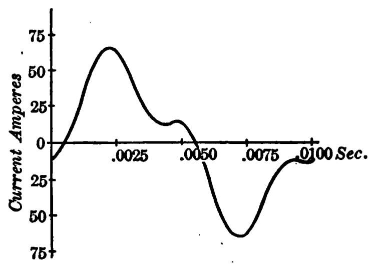

Problem 6, Chapter XIII



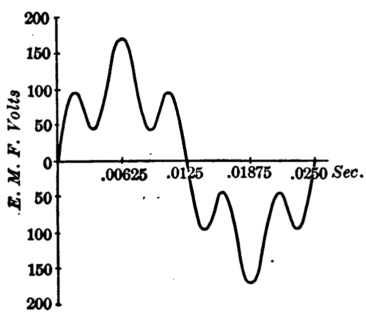

Problem 7, Chapter XI



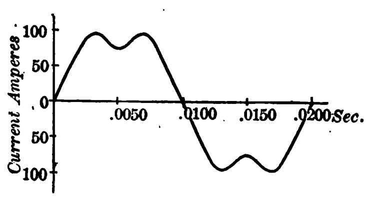

Problem 7, Chapter XIII



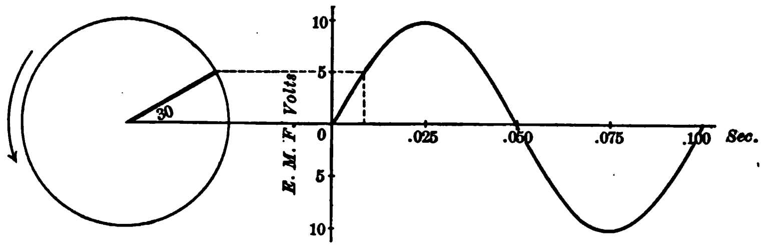

Problem 9, Chapter XIII



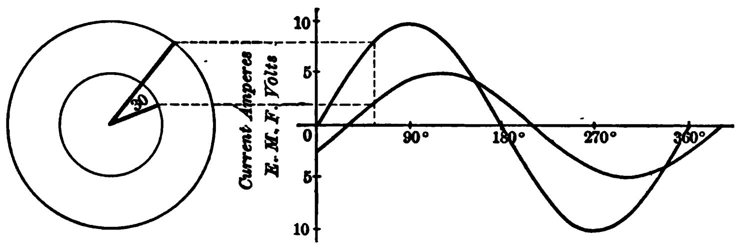

Problem 8, Chapter XIII



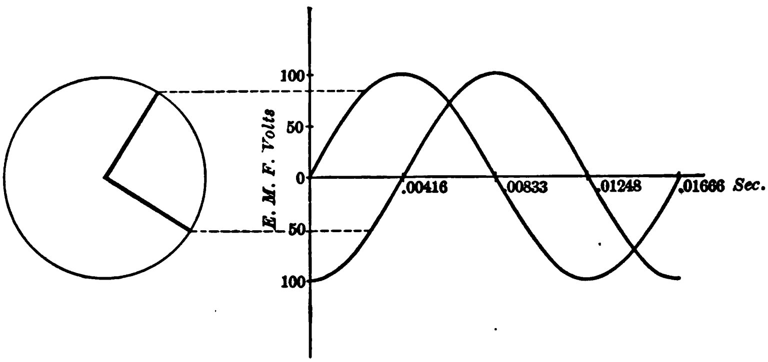

Problem 11, Chapter XIII



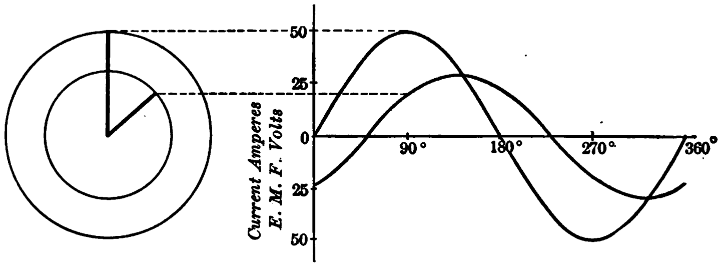

Problem 1, Chapter XIII



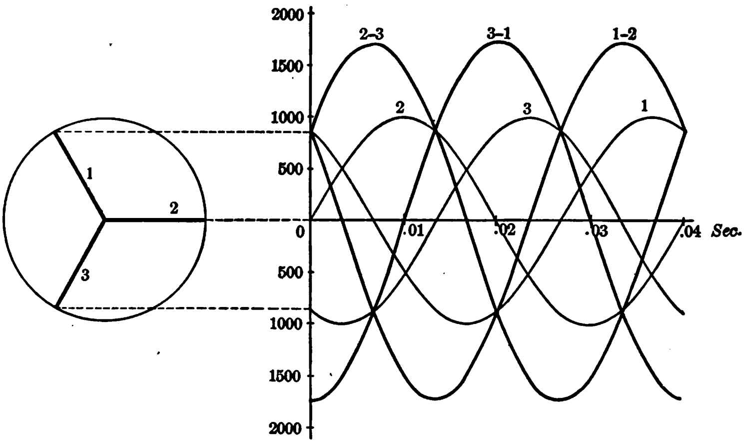

Problem 3, Chapter XIII



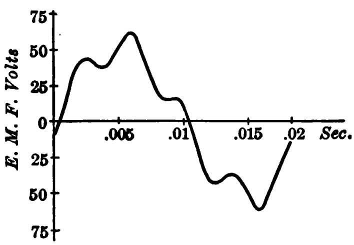

Problem 12, Chapter XIII



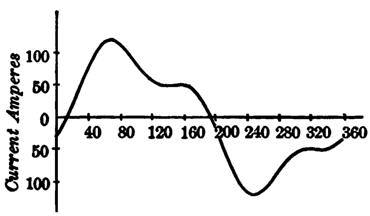

Problem 5, Chapter XIII



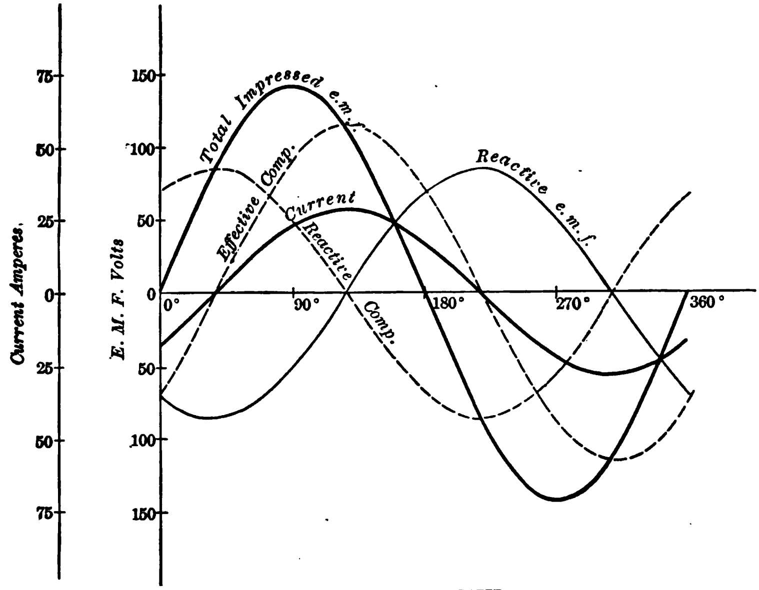

Problem 13, Chapter XIII



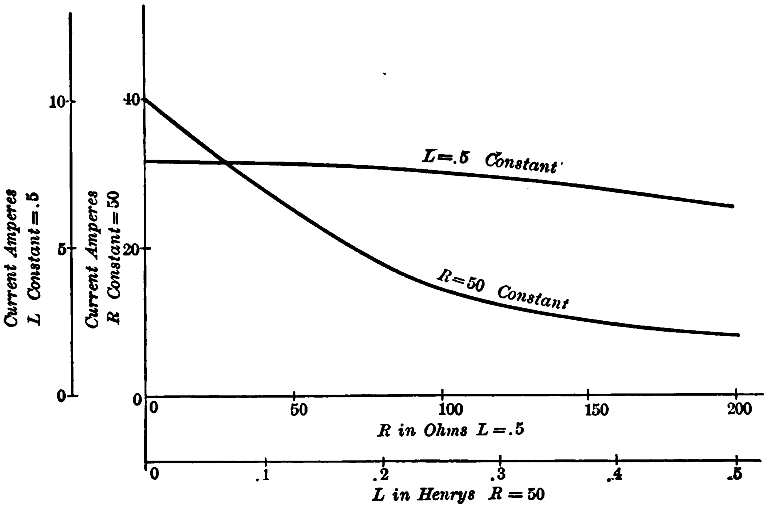

Problem 10, Chapter XIII



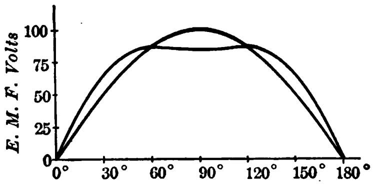

Problem 15, Chapter XIII



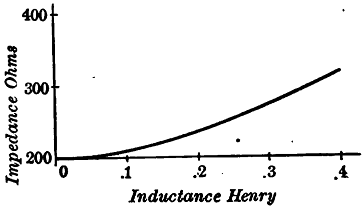

Problem 4, Chapter XVI



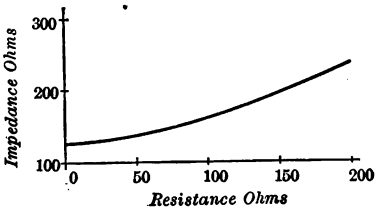

Problem 20, Chapter XVI



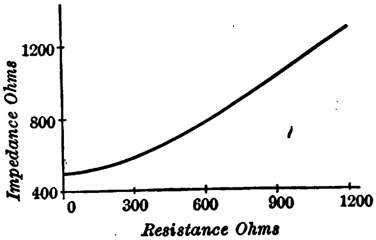

Problem 14, Chapter XIII



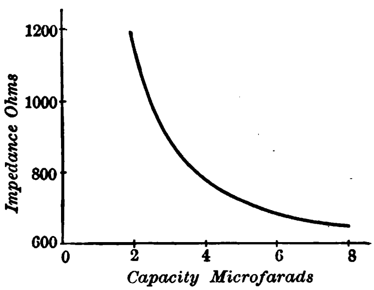

Problem 18, Chapter XVI



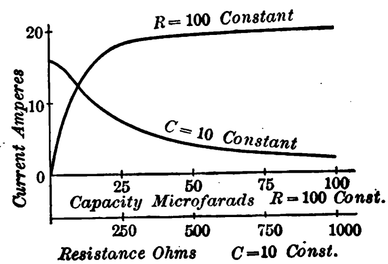

Problem 19, Chapter XVI



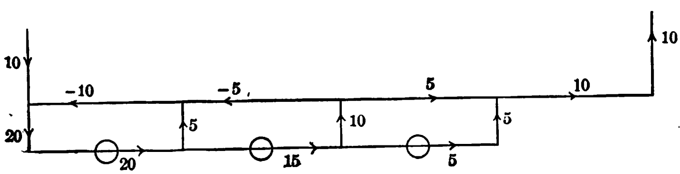

Problem 36, Chapter XVI



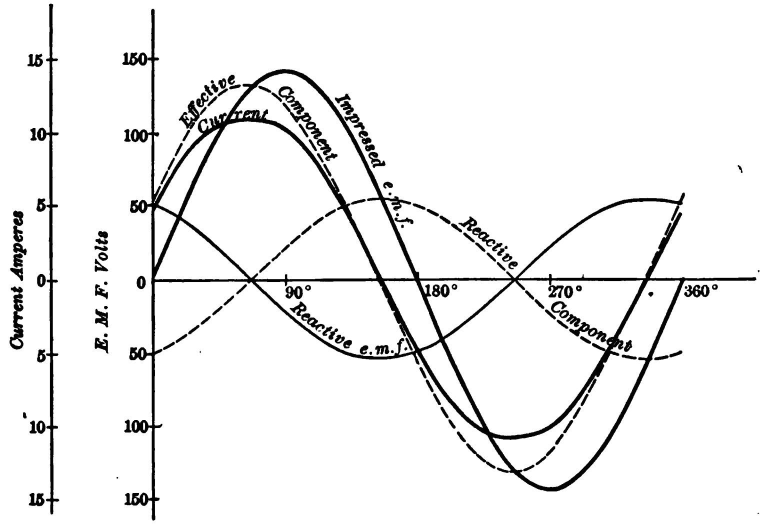

Problem 37, Chapter XVI



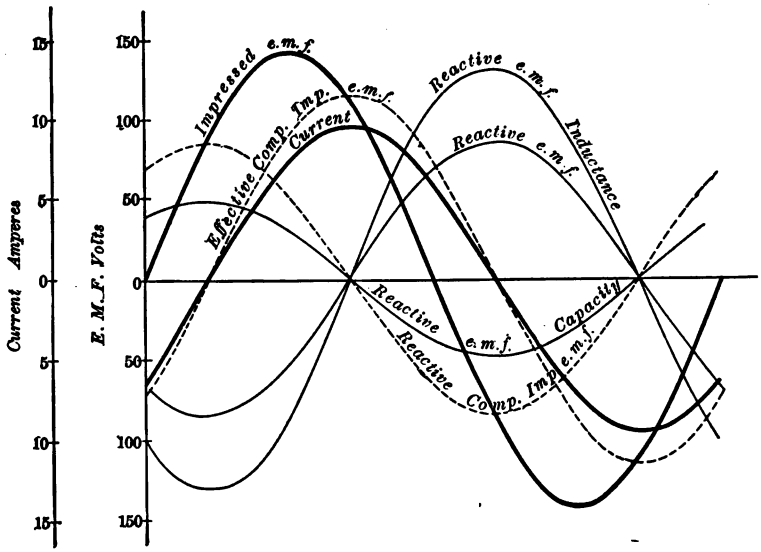

Problem 38, Chapter XVI



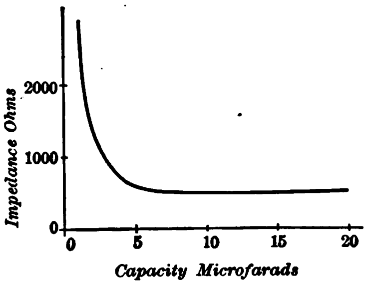

Problem 22, Chapter XXII



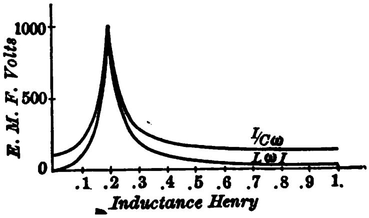

Problem 25, Chapter XVI



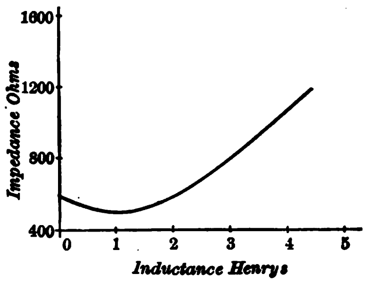

Problem 42, Chapter XVI



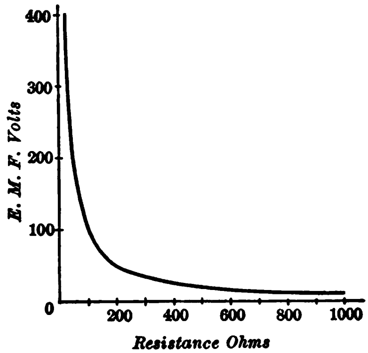

Problem 49, Chapter XVI



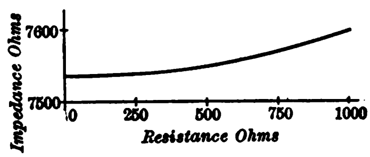

Problem 55, Chapter XVI



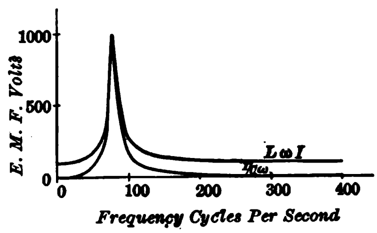

Problem 50, Chapter XVI



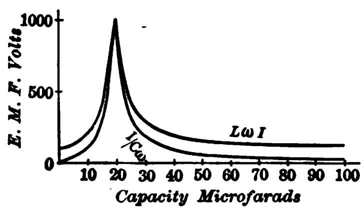

Problem 56, Chapter XVI



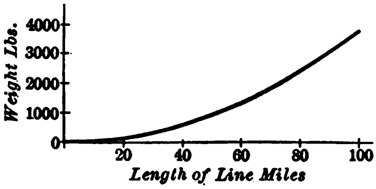

Problem 51, Chapter XVI



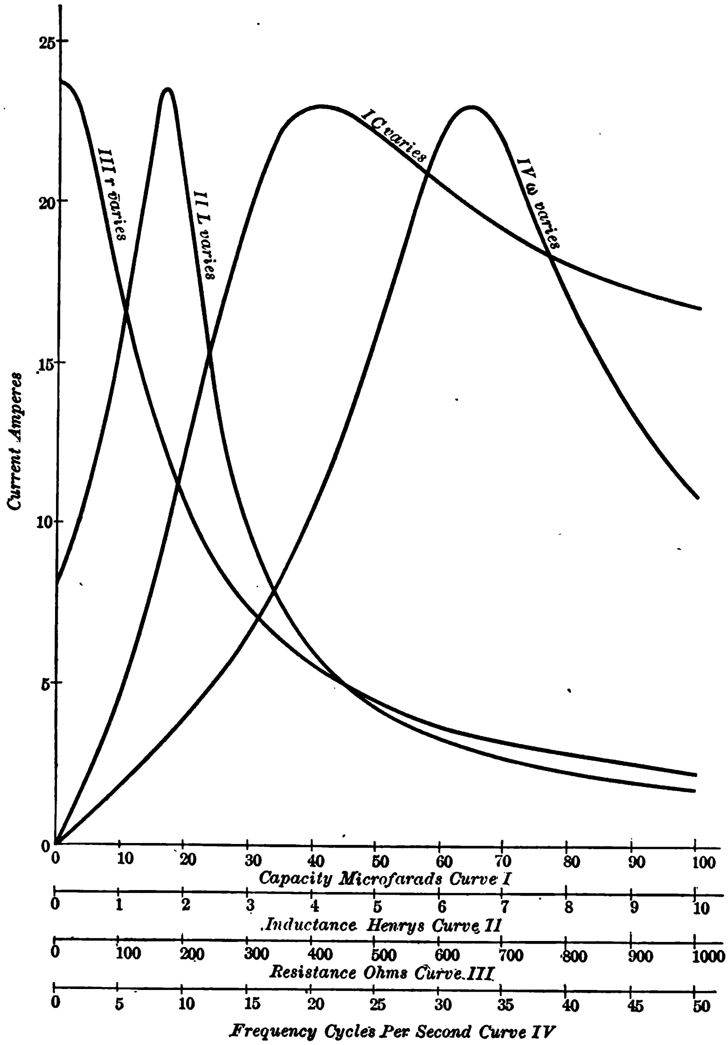

Problem 57, Chapter XVI



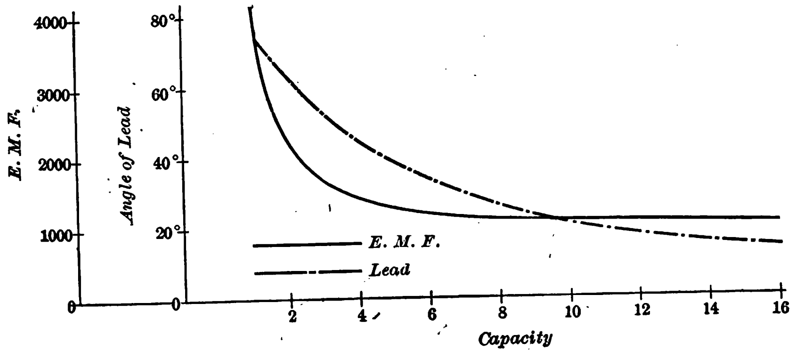

Problem 54, Chapter XVI



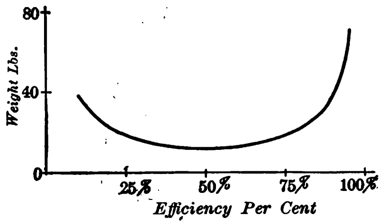

Problem 2, Chapter XXVI



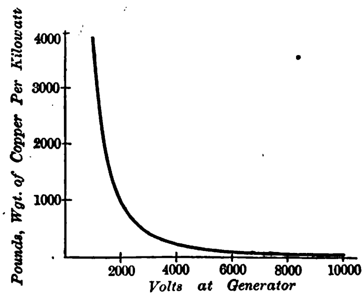

Problem 52, Chapter XVI



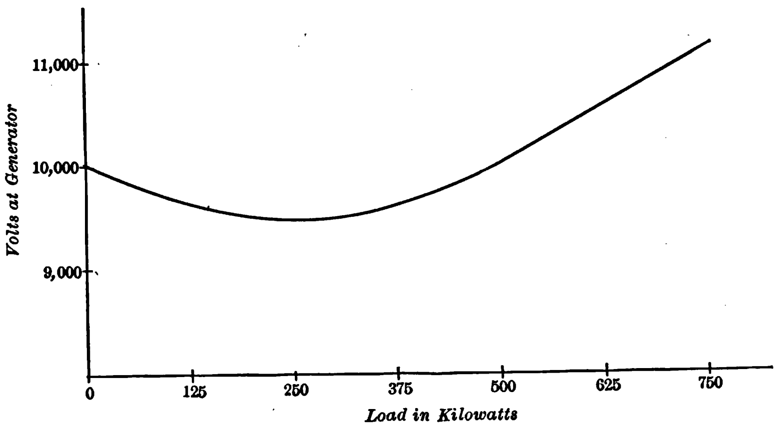

Problem 60, Chapter XVI



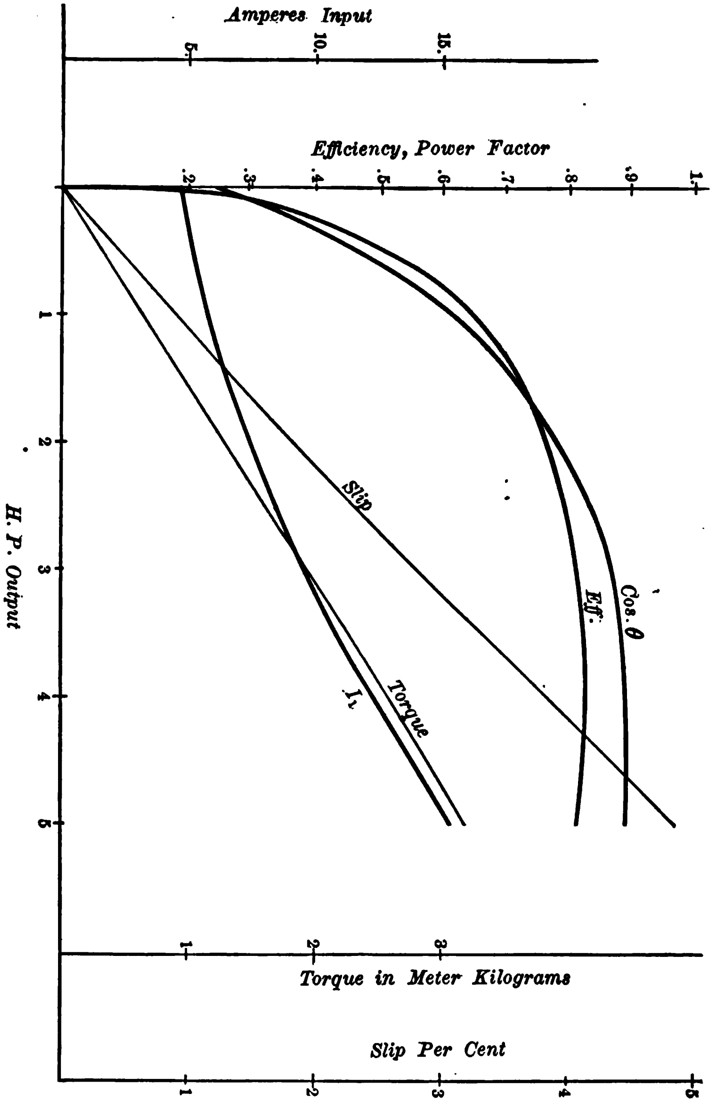

Problem 1, Chapter XXVI



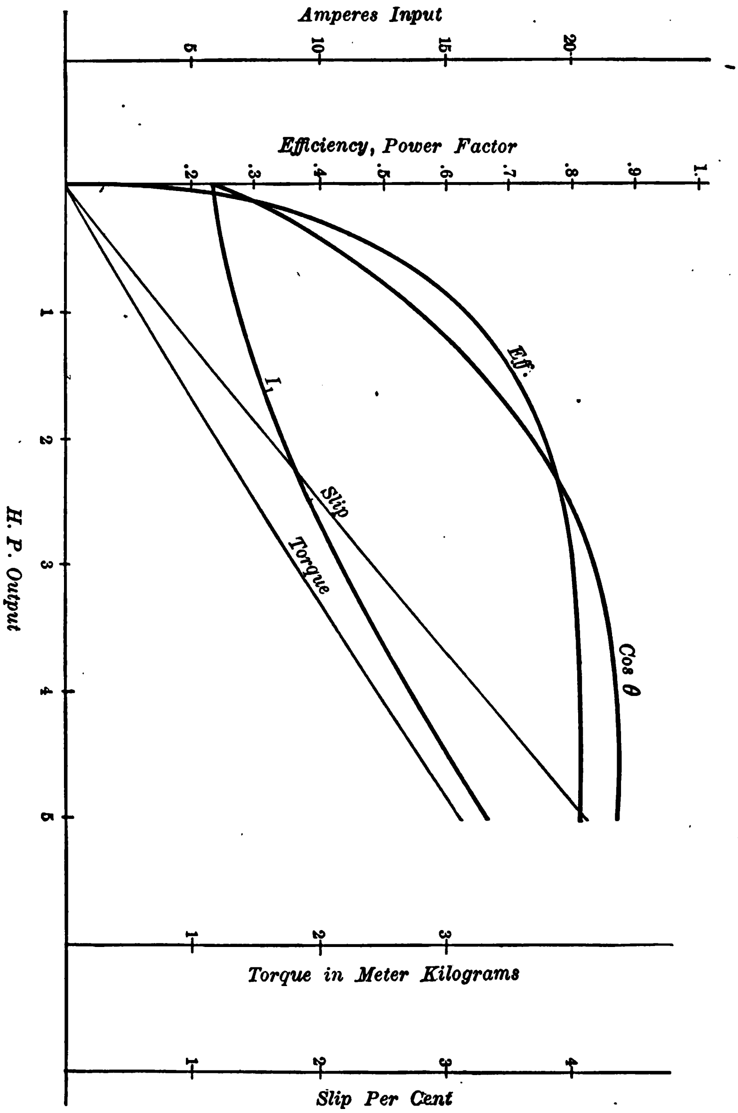

Problem 3, Chapter XXVI




Problem 13, Chapter XXVI




Problem 27, Chapter XXIV




Problem 28, Chapter XXIV